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PERMANENT VS. TEMPORARY FISCAL EXPANSION IN A TWO-SECTOR SMALL OPEN ECONOMY MODEL[1]

Olivier CARDI[2] and Romain RESTOUT[3]

Abstract

This contribution shows that the duration of a fiscal shock together with sectoral capital intensity matter in determining the dynamic and steady-state effects in an intertemporal-optimizing two-sector small open economy model. First, unlike a permanent shock, net foreign asset position always worsens in the long-run after a transitory fiscal expansion. Second, steady-state changes in physical capital depend on sectoral capital-labor ratios but their signs may be reversed compared to the corresponding permanent public policy. Third, investment and the current account may now adjust non monotonically. Fourth, a temporary fiscal shock always crowds-out (crowds-in) investment in the long-run whenever the non traded (traded) sector is more capital intensive.

Keywords: Nontraded goods; Temporary Shocks; Government Spending; Current Account.

JEL Classification: F41, E62, E22, F32.

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1 Introduction

In the last fifteen years, a large literature has investigated the short-term and long-term effects of government spending shocks within dynamic general equilibrium frameworks. Most of these papers use a one-sector model with elastic labor supply and analyze both permanent and temporary fiscal policies, in particular Baxter and King [1993] and Karayalçin [1999] who assume perfect competition, Devereux et al. [1996] and Heijdra and Ligthart [2007] who consider monopolistic competition. Our contribution is to investigate the dynamic and steady-state effects of fiscal expansions in a two-sector small open economy model by differentiating analytically between permanent and temporary policies on the one hand and between short-lived and long-lived transitory spending shocks on the other.

In a one-sector small open economy model with capital adjustment costs and elastic labor supply, an unanticipated permanent rise in government spending generates a *wealth effect* which lowers consumption (and leisure) and raises labor supply, stimulates capital accumulation (due higher capital marginal productivity) and deteriorates the current account. If the fiscal policy is solely implemented transitorily, short-term and long-term impacts remain qualitatively similar, although they are much more moderated (see e. g. Karayalçin [1999], Schubert and Turnovsky [2002]). In our paper, we make the distinction between traded and non traded goods and we abstract from labor-leisure trade-off for reason of analytical simplicity. The formal study of transitory changes leads to some striking results since we show that the short-term and more importantly the long-run changes of key economic variables may be reversed compared to permanent government expenditure shocks. These conclusions stand in sharp contrast to the predictions of small open country models restricted to a one-sector framework.

We used a two-sector model similar to that developed in Turnovsky and Sen [1995] in which a traded and a non traded good are produced and consumed in the economy. The introduction of non tradables allows to capture several important common features of industrialized countries and to enlarge the effects at work. First, empirical works indicate that the non traded sector accounts for about 60% of GDP in developed countries.[1] Second, data underline that government spending tends to fall heavily on non traded goods.[2] The distinction between traded and non traded goods is crucial since short-run and long-run behavior of the capital stock and net foreign asset position depend on relative sectoral capital intensities. In addition, while a rise in government spending falling on the traded good influences capital accumulation (and thus GDP) indirectly through the standard *wealth effect*, an increase in public expenditure falling on non tradables gives rise to a *real interest rate effect* induced by the expected change in the real exchange rate which works against the former effect.[3]

Our study departs from Turnovsky and Sen's [1995] analysis in three respects. First, while Turnovsky and Sen focus on demand and supply permanent shocks, most of our attention is devoted to the case of temporary fiscal policies, with the purpose to draw out the striking differences with the macroeconomic adjustment after a permanent fiscal expansion. This extension is crucial since the existence of the zero-root property implies that temporary shocks have permanent effects and more importantly a temporary public policy is not a dampening down of the permanent effect of the corresponding permanent policy.[4] Second, we use a dual approach to deal with consumption decisions which in turn allows for bringing out more clearly the pivotal factors at stake in determining the macroeconomic dynamics. Third, we calibrate and simulate the model. Numerical exercises point out the role of the duration of the public policy by computing the impulse functions for different times of policy removal, and by providing quantitative information about the sizes of the short-term and long-term changes of key economic variables.

To conduct the analysis of temporary shocks, we apply the *two-step* approach proposed by Schubert and Turnovsky [2002] to the zero-root property. Their *two-step* approach has an attractive use since it allows to express the steady-state values of macroeconomic aggregates as functions of the marginal utility of wealth and policy parameters and to estimate in a rigorous and clear way their long-run changes after a temporary fiscal expansion. This paper can be viewed as a pursuit of the theoretical work initiated by Schubert and Turnovsky [2002] by considering a two-sector model, and a complementary analysis to the recent continuous-time intertemporal-optimizing literature investigating the effects of temporary fiscal shocks, in particular Karayalçin [1999] and Heijdra and Ligthart [2007], who restrict their model to a one-sector economy.

Our main results are as follows. First, unlike two-sector models by Obstfeld [1989] and Turnovsky and Sen [1995] where the economy adjusts monotonically after a permanent fiscal shock, the temporary nature in the shift of government purchases may give rise to hump- or U-shaped transient paths for investment and the current account. In particular, investment may now be crowded-in or crowded-out in the short-run depending on the duration of the public policy. Second, different from permanent fiscal policies whenever the rise in government spending is temporary, the net foreign asset position always worsens in the long-run, irrespective of the relative capital intensity or the good on which the rise in public spending falls. The third result is perhaps the most startling. If the transitory rise in government purchases falls on the non traded good, the steady-state changes in physical capital stock and international assets holding are reversed compared to those derived after a permanent policy when the traded sector is more capital intensive. With the reversal capital intensity, the capital stock falls in the long-run instead of rising. Fourth, while a temporary fiscal expansion always crowds-in investment in the long-run whenever the traded sector is more capital intensive, the capital stock is decumulated if the non traded sector features a higher capital-labor ratio, irrespective on the good on which public spending falls. Fifth, we show formally that a temporary fiscal expansion is not a dampening down of the permanent effect of the corresponding permanent policy.

The paper is organized as follows. Section 2 presents the framework of a two-sector small open economy. In section 3, we analyze the equilibrium dynamics and the steady-state of the model. Sections 4 and 5 compare the short-term and long-term effects of unexpected permanent and temporary rise in government spending falling on the traded good and the non traded good. Transitional paths and responses of key macroeconomic variables are computed numerically in section 6. Section 7 concludes and provides direction for further research.[5]

2 The Framework

Consider a small open economy that is populated by a constant number of identical households and firms that have perfect foresight and live forever. The country is small in world goods and capital markets and faces given world interest rate, $r^{\star}$. There are two types of goods. The representative firm produces both a traded good that can be exported and a non traded good which is dedicated to capital accumulation and consumption. The traded good is chosen as numeraire.

Households

At each instant the representative agent consumes traded goods and non traded goods denoted by c^T and c^N. The measure of utility of consumption at time t, $c(t)$, is given by $c(t) = c\left(c^T(t), c^N(t)\right)$ with $c(.)$ a positive, increasing, concave and linearly homogeneous aggregator function. The representative household maximizes a lifetime utility function:

$$\int_0^\infty u\left[c\left(c^T(t), c^N(t)\right)\right] e^{-\delta t} \mathrm{d}t, \tag{1}$$

where δ is the consumer's discount rate and marginal utility from real consumption is positive and decreasing, that is $u_c > 0$, $u_{cc} < 0$.

Since $c\,(.)$ is homothetic, the household's maximization problem can be decomposed into two stages. At the **first stage**, the household minimizes the cost, $E(t) = c^T(t) + p(t)c^N(t)$, for a given level of subutility, $c(t)$, where $p(t)$ is the relative price of the non traded good (or the real exchange rate). The assumption that the subutility function $c\,(.)$ is linear homogeneous implies that the total expense in consumption goods can be expressed as $E(t) = p_c\,(p(t))\,c(t)$. The ratio E/c gives the unit cost function dual to c (or consumption-based price index), denoted by $p_c\,(p)$, with $p_c' > 0$, and $p_c'' < 0$. Intra-temporal allocations between non traded goods and traded goods follow from Sheppard's Lemma (or the envelope theorem): $c^N = p_c'\,(p)\,c$ and $c^T = \left[p_c\,(p) - pp_c'\,(p)\right] c$.[6]

Households inelastically supply one unit of labor services ($L(t) = 1$) for which they receive the wage $w(t)$. Their non-human wealth, denoted by $a(t)$, measured in terms of the traded good is equal to the sum of internationally traded bonds holding, $n(t)$, and the value of domestic capital stock, $p(t)K(t)$, expressed in terms of the traded good. Since these assets are perfect substitutes, they earn the same rate of return. In the **second stage**, consumers choose their real consumption, c, and the rate of accumulation of financial wealth, $\dot{a}(t)$, to maximize (1) subject to the flow budget constraint,

$$\dot{a}(t) = r^\star a(t) + w(t) - p_c\,(p(t))\,c(t) - Z, \tag{2}$$

and initial condition $a(0) = a_0$. Households' income consists of interest earnings on traded bonds holding, $r^\star n(t)$, rental income $r^\star pK$, and labor revenue, w. They pay lump-sum taxes, denoted by Z, to the government and spend an amount equal to $p_c c$ in consumption goods purchases.

Firms

Perfectly competitive firms produce a traded (non traded) good, index by superscript T (N), using capital, K^T (K^N), and labor, L^T (L^N), according to a constant returns to scale production function, $Y^T = F\left(K^T, L^T\right)$ ($Y^N = H\left(K^N, L^N\right)$), which is assumed to have the usual neoclassical properties of positive and diminishing marginal products. The production of the traded good can be consumed domestically or exported. The non traded good may be used for capital accumulation and consumption.

For analytical convenience, we assume that the production choices facing the traded and non traded sectors can be consolidated. The representative firm maximizes the present value of anticipated future cash flow discounted at the world interest rate:

$$\int_0^{\infty} \left[F\left(K^T, L^T\right) + pH\left(K^N, L^N\right) - pI - w\right] e^{-r^{\star}s} \mathrm{d}s, \tag{3}$$

subject to capital accumulation,[7]

$$I = \dot{K}, \tag{4}$$

together with the labor and capital intersectoral allocation constraints,

$$L^T + L^N = 1, \quad K^T + K^N = K, \tag{5}$$

and the initial condition for capital stock $K(0) = K_0$.

Government

The final agent in the economy is the government who finances government expenditure by raising lump-sum taxes Z in accordance with the balanced condition:

$$g^T + p(t)g^N = Z. \tag{6}$$

Public spending consists in purchases of traded goods, g^T, and non traded goods, g^N.

2.1 Macroeconomic Equilibrium

Denoting by $k^i \equiv K^i/L^i$ the capital-labor ratio for sector $i = T, N$, we express the production functions in intensive form, that is $y^T = f\left(k^T\right) \equiv F\left(K^T, L^T\right)/L^T$ and $y^N = h\left(k^N\right) \equiv H\left(K^N, L^N\right)/L^N$.

First-Order Conditions

To obtain the macroeconomic equilibrium, we first derive the optimality conditions for households and firms and combine these with the inputs allocation constraints and accumulation equations. The macroeconomic equilibrium is summarized by the following set of equations:

$$u_c(c) = p_c(p)\lambda, \tag{7a}$$

$$f_k\left(k^T\right) = ph_k\left(k^N\right) \equiv r^K, \tag{7b}$$

$$\left[f\left(k^T\right) - k^T f_k\left(k^T\right)\right] = p\left[h\left(k^N\right) - k^N h_k\left(k^N\right)\right] \equiv w, \tag{7c}$$

$$L^T k^T + L^N k^N = K, \tag{7d}$$

$$L^T + L^N = 1, \tag{7e}$$

$$\dot{\lambda} = \lambda\left(\delta - r^{\star}\right), \tag{7f}$$

$$\dot{p} = p\left[r^{\star} - h_k\left(k^N\right)\right], \tag{7g}$$

$$\dot{K} = L^N h\left(k^N\right) - p_c'(p)\, c - g^N, \tag{7h}$$

$$\dot{n} = r^{\star} n(t) + L^T f\left(k^T\right) - \left(1 - \alpha_c\right) p_c(p)\, c - g^T, \tag{7i}$$

and the transversality conditions

$$\lim_{t \to \infty} \lambda a(t) e^{-r^{\star} t} = \lim_{t \to \infty} p(t) K(t) e^{-r^{\star} t} = \lim_{t \to \infty} n(t) e^{-r^{\star} t} = 0, \tag{8}$$

where λ is the co-state variables associated with dynamic equation (2). The last equality of (8) rules out the possibility for the small country of running up infinite debt or credit and ensures that the nation remains intertemporally solvent.

The static efficiency condition (7a) requires that along an optimal path the marginal current utility of real consumption is equal to the marginal utility of wealth in the form of internationally traded bonds measured in terms of the non traded good, $p_c\lambda$. Equations (7b) and (7c) require that in equilibrium, the rate of return on capital and labor equalize in the traded and non traded sectors. These two conditions will determine in turn the sectoral capital-labor ratios which together with (7d)-(7e) determinate labor sectoral allocation. Equation (7g) requires that in equilibrium, the rate of return on capital used in the non traded sector consisting of its marginal product, h_k, must equal the rate of return on traded bonds, i. e. the domestic real interest rate equals to the world interest rate less the rate of appreciation of the real exchange rate, p. Equation (7h) described the non traded good market clearing condition according to which the non traded output may be dedicated both to consumption of households and government and to capital accumulation. To determine the accumulation equation of foreign bonds (7i), we differentiated expression $n = a - pK$ with respect to time, then we substituted dynamic equations

(2), (4) and (7h). Equation (7i) is equal to the current account surplus, which comprises the trade balance, equal to the value of output less absorption, plus interest payments on outstanding assets.

Finally, with a constant rate of time preference and an exogenous world interest rate, we require the time preference rate to be equal to the world interest rate:

$$\delta = r^{\star}, \tag{9}$$

in order to generate an interior solution. This standard assumption made in the literature implies that the marginal utility of wealth, λ, must remain constant over time and is always at its steady state level, $\bar{\lambda}$. This gives rise to the zero-root property which in turn implies that temporary perturbations have permanent effects.

Short-Run Static Solutions

The static efficiency condition (7a) can be solved for the short-run macroeconomic equilibrium which of course must hold at any point of time. We obtain:

$$c = c\left(\bar{\lambda}, p\right), \quad \text{with} \quad c_{\bar{\lambda}} < 0, \quad c_p < 0. \tag{10}$$

A higher marginal utility of wealth, measured in terms of the traded good, $\bar{\lambda}$, encourages savings and induces the representative agent to reduce the consumption of both goods. Given the marginal utility of wealth, a real exchange rate appreciation (i. e. a rise in p) raises the marginal utility of wealth measured in terms of the non traded good through the rise in the consumption price index and leads to a fall in real consumption.

From static optimality conditions (7b)-(7e), we may solve for sector capital intensities ratios and for labor

$$k^T = k^T(p), \quad k^N = k^N(p), \quad L^T = L^T(K,p), \quad L^N = L^T(K,p). \tag{11}$$

where $L_p^T < 0$ and $L_p^N > 0$ and the signs of other partial derivatives are dependent upon relative sectoral capital intensities. An appreciation in the real exchange rate attracts resources from the traded to the non traded sector. If $k^N > k^T$, labor increases in relative abundance which lowers the wage-rental ratio, w/r^K, and consequently encourages the producers in the two sectors to substitute labor for capital, causing a decline in the capital-labor ratios, k^T and k^N. From the resource constraints, labor moves from the traded sector to the non traded sector and consequently raises the non traded output. Finally, a higher aggregate capital stock K induces a shift of labor in the direction of the more capital intensive sector.[8]

3 Equilibrium Dynamics and the Steady-State

Equilibrium Dynamics

Inserting short-run static solutions (10) and (11) into (7g) and (7h), linearizing these equations around the steady-state, we obtain a second-order linearized system which possesses one negative eigenvalue and one positive eigenvalue with:

$$\mu_1 < 0 < r^\star < \mu_2. \tag{12}$$

Since the system features one state variable, K, and one jump variable, p, the equilibrium yields a unique one-dimensional stable saddle-path, irrespective of the sectoral capital intensity.[9] Starting from an initial aggregate capital stock $K(0) = K_0$, the stable paths for K and p write as follows:

$$K(t) = \tilde{K} + \left(K_0 - \tilde{K}\right) e^{\mu_1 t}, \quad p(t) = \tilde{p} + \omega_2^1 \left(K(t) - \tilde{K}\right), \tag{13}$$

where $\omega_2^1 = 0$ if $k^T > k^N$ and $\omega_2^1 < 0$ if $k^N > k^T$; we denoted steady-state values with a tilde. As emphasized by Turnovsky and Sen [1995], a striking feature of the transitional path of the real exchange rate is its dependency upon the relative capital intensities of sectors T and N.

If the capital intensity of the non traded sector exceeds that of the traded sector, the slope of the stable path is negatively sloped in the (K, p)-space. Intuitively, a higher demand for the non traded good causes an immediate real exchange rate appreciation (p jumps upward), labor shifts in direction of the non traded sector, output of that sector rises and thus aggregate capital stock starts growing. Since a change in demand leaves unchanged the steady-state value of the relative price of non tradables, the real exchange rate must depreciate along the stable trajectory as physical capital accumulates. This expected depreciation raises the consumption-based real interest rate ($r^c = r^\star - \alpha_c \frac{\dot{p}}{p}$) above the time preference rate δ which in turn provides an incentive to reallocation expenditure towards future.[10]

Withe the reversal capital intensity ($k^T > k^N$), the dynamics of the relative price degenerate. For example, a higher demand for non traded appreciates instantaneously the real exchange rate which leads to a rise in the capital-labor ratios, k^T and k^N. The relative price must fall to reach its unchanged long-run equilibrium, i. e. $\dot{p}(t)/p(t) < 0$. But in the same time, the rental rate on capital h_k decreases because k^N is higher. The no-arbitrage condition (7g) does no longer hold and the economy moves along an unstable path. Since p must be fixed, equality (9) implies a flat transitional temporal for real consumption.

Inserting short-run static solutions into (7i), linearizing around the steady-state, substituting solutions for $K(t)$ and $p(t)$, and invoking the transversality condition (8), we obtain the stable solution for $n(t)$ consistent with long-run solvency:

$$n(t) - \tilde{n} = \frac{N_1}{\mu_1 - r^\star} \left(K(t) - \tilde{K} \right). \tag{14}$$

where $N_1 / (\mu_1 - r^\star)$ is unambiguously negative if $k^T > k^N$ ($N_1 / (\mu_1 - r^\star) = -\tilde{p}$) but its sign remains undetermined if $k^N > k^T$ ($N_1 / (\mu_1 - r^\star) = -\tilde{p} \left(1 + \frac{1}{\mu_2} \frac{\tilde{c}^N}{\tilde{p}} \sigma_c \omega_2^1 \right)$). According to empirical studies which suggest a negative relationship between the current account, $\dot{n}(t)$, and investment in capital $\dot{K}(t)$ (see Glick and Rogoff [1995]), we assume that $N_1 / (\mu_1 - r^\star) < 0$, irrespective of the sectoral capital intensity.

Finally, linearization of (2) in the neighborhood of the long-run equilibrium together with transversality conditions (8), keeping in mind that the wage rate is a function of the real exchange rate ($w = w(p)$), enables us to derive the stable solution for the financial wealth:

$$a(t) = \tilde{a} + \frac{M_1}{\mu_1 - r^\star} \left(p(t) - \tilde{p} \right), \tag{15}$$

where $M_1 = -\left(\tilde{K}\mu_2 - \sigma_c \tilde{c}^N \right) < 0$ if $\sigma_c < 1$ (as empirical studies suggest). If $k^T > k^N$, the current account offsets exactly the investment flow, $ca(t) = -\tilde{p}I(t)$. The savings flow $S(t) = \dot{a}(t) = 0$ and thus financial wealth, remains unaffected, i. e. $a(t) = \tilde{a}$. Unlike, if $k^N > k^T$, the current account mirrors both savings and investment behavior, which themselves are determined by consumption and production decisions. From (15), savings and real exchange rate depreciation are positively correlated along the stable branch. This is intuitive. Consider, for example, that the real exchange rate appreciates on impact following a greater demand for non tradable goods. A higher $p(t)$ lowers the wage rate ($w_p < 0$ if $k^N > k^T$), and raises both total expense in consumption goods and lump-sum taxes Z (because government spending in non traded measured in terms of the traded good, i. e. pg^N rises) and consequently leads to a negative savings flow.[11]

Steady-State

The steady-state of the economy is obtained by setting $\dot{K} = \dot{p} = \dot{n} = 0$ and is defined by the following set of equations:

$$h_k \left[k^N (\tilde{p})\right] = r^\star, \tag{16a}$$

$$Y^N \left(\tilde{K}, \tilde{p}\right) - p_c' (\tilde{p})\, c \left(\bar{\lambda}, \tilde{p}\right) - g^N = 0, \tag{16b}$$

$$r^\star \tilde{n} + Y^T \left(\tilde{K}, \tilde{p}\right) - (1 - \alpha_c)\, p_c (\tilde{p})\, c \left(\bar{\lambda}, \tilde{p}\right) - g^T = 0, \tag{16c}$$

and the intertemporal solvency condition

$$(n_0 - \tilde{n}) = \frac{N_1}{\mu_1 - r^\star} \left(K_0 - \tilde{K}\right). \tag{16d}$$

We have expressed traded and non traded capital-labor ratios as functions of the real exchange rate from equations (7b)-(7c) holding at each instant of time. The steady-state equilibrium defined by these four equations jointly determine $\tilde{p}$, $\tilde{K}$, $\tilde{n}$, $\bar{\lambda}$.

Equation (16a) asserts that the rate of change of the real exchange rate is zero when the marginal physical product of capital in the non traded sector is equal to the exogenous world interest rate, $r^\star$. Because the world interest rate is fixed, demand disturbance taking the form of government expenditure shocks leaves unchanged the steady-state values of the real exchange rate, $\tilde{p}$, and the capital-labor ratios, $\tilde{k}^T$ and $\tilde{k}^N$. Equation (16b) restates that the production of non traded goods must be exactly outweighed by a demand counterpart for a zero investment and determines $\tilde{K}$ for given $\bar{\lambda}$. Equation (16c) implies that the current account must be zero in the long-run and determines $\tilde{n}$ for given $\bar{\lambda}$. Finally, the linearized version of the nation's intertemporal budget constraint (16d) implies that the steady-state depends on the initial stocks K_0 and n_0 and determines the equilibrium value of the marginal utility of wealth.

One characteristic of the model merits comment. System (16) may be solved for the steady-state values by applying a *two-step* procedure. Using equations (16a) to (16c), $\tilde{K}$ and $\tilde{n}$ can be expressed as functions of marginal utility of wealth $(\bar{\lambda})$ government spending on the traded (g^T) and the non traded good (g^N). This yields to the following functions:[12]

$$\tilde{K} = K \left(\bar{\lambda}, g^N\right), \quad \tilde{n} = v \left(\bar{\lambda}, g^T, g^N\right), \tag{17}$$

with $K_{\bar{\lambda}} \lesseqgtr 0$ and $K_{g^N} \gtreqless 0$ depending on wether $k^N \gtrless k^T$, and $v_{\bar{\lambda}} < 0$, $v_{g^T} > 0$, $v_{g^N} > 0$. In the second step, we insert these functions into the economy's intertemporal budget constraint (i. e. eq (16d)), which may be solved for the equilibrium value of the marginal utility of wealth:

$$\bar{\lambda} = \lambda \left(K_0, n_0, g^T, g^N\right), \quad \lambda_K < 0, \quad \lambda_n < 0, \quad \lambda_{g^T} > 0, \quad \lambda_{g^N} > 0. \tag{18}$$

From (18), the equilibrium value of the marginal utility of wealth depends on K_0 and n_0. Consequently, the steady-state depends on initial conditions and therefore temporary shocks have permanent effects. When the disturbance lasts solely $\mathcal{T}$ periods, we must take into account that the stocks of capital and traded bonds have been accumulated over the unstable period $(0, \mathcal{T})$. This implies new initial conditions for the small open economy once the public policy is removed, say $K_{\mathcal{T}}$ and $n_{\mathcal{T}}$, which in turn influences the new steady state.

4 Permanent vs. Temporary Permanent Increase in Government Spending on Traded Good

In this section, we explore the transitional and long-run effects of a rise in government expenditure on the traded good, g^T. We conduct our analysis by studying an unanticipated permanent increase (denoted by the subscript $_{perm}$) and an unanticipated temporary rise (denoted by the subscript $_{temp}$) in g^T.

For the government budget to be balanced at the steady-state, an increase in public spending must be financed through higher taxes. Therefore, the present value of wealth is reduced. Due to the zero-root property, the marginal utility of wealth jumps instantaneously upward to its new steady-state value, $\bar{\lambda}$, and remains constant from thereon. Since the long-run relative price remains unchanged (and therefore the consumption price index), the *wealth effect* induces agents to lower their steady-state real consumption level.

An Unanticipated Permanent Rise in g^T

Long-run changes in the aggregate capital stock and net foreign position depend on the relative sectoral capital intensity. More specifically, the long-term variation in the steady-state value of K plays the key role of clearing the non tradables market. Irrespective of the relative capital-labor ratio values between sectors, the once-for all increase in $\bar{\lambda}$ lowers consumption of non tradables c^N which calls for a long-term fall in output Y^N to eliminate the excess supply. If the non traded (traded) good sector is relatively more capital intensive, the steady-state value of the capital stock will fall (increase) in order to meet the lower demand for non traded goods. The long-run current account equilibrium requires a rise (decrease) in the stock of internationally traded bonds holding since the trade balance worsens (improves).

The transitional paths are depicted in figure 2(a) in the case $k^T > k^N$ and in figure 2(b) in the case $k^N > k^T$. If $\underline{k^N > k^T}$, an unanticipated permanent increase in g^T raises the demand for traded goods and leads to an initial depreciation in the real exchange rate, $\mathrm{d}p(0) < 0$. The positive influence on real consumption originating from the fall in p dampens the *wealth effect*. Real consumption decreases on impact, but by less than in the long-run.[13] Regarding the supply-side, the initial drop in the relative price induces a shift of employment from the non traded sector to the traded sector. The fall of labor reduces in turn the production in the non traded sector which calls for a decline in investment because the non traded market must clear. Along the stable path, real consumption and capital stock decrease gradually and the stock of foreign assets rises. If the intertemporal elasticity of substitution for consumption is less than one, as empirical evidence suggests, the small country reaches the long-run equilibrium with greater financial wealth.

If $\underline{k^T > k^N}$, the real exchange rate remains unaffected and the fall in c^N releases some resources for investment. Since the capital stock shifts towards the sector which is relatively more capital intensive, output in the traded sector rises. Along the stable adjustment, aggregate capital accumulation is exactly matched by a proportional current account deficit triggered by the rise in g^T. Consequently, the steady-state value of the financial wealth remains unaffected.

An Unanticipated Temporary Rise in g^T

We turn now to the study of an unanticipated temporary fiscal expansion. We suppose that government spending rises unexpectedly at time $t = 0$ from the original level g_0^T to level g_1^T, over the period $0 \leq t < \mathcal{T}$, and revert back at time $\mathcal{T}$ permanently to its initial level $g_{\mathcal{T}}^T = g_2^T = g_0^T$. The temporary nature of the fiscal shock requires to consider two periods, say period 1 ($0 \leq t < \mathcal{T}$) during which the small country follows unstable paths and period 2 ($t \geq \mathcal{T}$) over which the economy converges towards its new long-run equilibrium. Compared to a permanent spending shock, we find that the current account may adjust non-monotonically. More importantly, temporary fiscal shocks always deteriorate the net foreign asset position in the long-run, irrespective of the sectoral capital intensity (and the good on which the fiscal expansion falls, say g^T or g^N).

Regardless of the sectoral capital intensity, $\bar{\lambda}$ jumps upward:

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp} = \lambda_{g^T}\left(1 - e^{-r^\star \mathcal{T}}\right) > 0. \tag{19}$$

Since $\lambda_{g^T} > 0$ denotes the steady-state change of $\bar{\lambda}$ for a permanent variation in g^T, we see that the change of $\bar{\lambda}$ for a temporary rise in g^T is smaller but of the same direction. This is quite intuitive since the *wealth effects* induced by the transitory rise in government spending extend over successively shorter periods as the persistence of the fiscal expansion diminishes.

Because government spending g^T returns to its initial level at the time the policy is removed, the steady-state changes of macroeconomic aggregates are solely driven by the once-for-all jump of $\bar{\lambda}$. Due to the absence of a direct effect (i. e. $K_{g^T} = 0$), the long-run behavior of physical capital is qualitatively similar to the long-run adjustment after a permanent fiscal expansion. Unlike, the steady-state change in net foreign asset position is reversed in the case $k^N > k^T$ compared to a permanent expansionary budget policy:

$$\left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^T}\right|_{temp} = v_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp} < 0 < \left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^T}\right|_{perm} = v_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{perm} + v_{g^T}. \tag{20}$$

The explanation is straightforward. Whenever the policy is transitory, government spending is restored back to its initial level at time $\mathcal{T}$, net exports improve unambiguously at the new steady-state which in turn requires a long-run fall in the stock of international assets.

To analyze the initial response of the investment flow, it is convenient to evaluate the linearized version of the market-clearing condition at time $t = 0$ and differentiate this with respect to g^T:

$$\left.\frac{\mathrm{d}I(0)}{\mathrm{d}g^T}\right|_{temp} = \left(Y_p^N - c_p^N\right) \left.\frac{\mathrm{d}p(0)}{\mathrm{d}g^T}\right|_{temp} + \sigma_c \frac{\tilde{c}^N}{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp}, \tag{21}$$

where $\left(Y_p^N - c_p^N\right) > 0$. The first term on the right-hand side of (21) which features the *relative price effect* turns to be null if $\underline{k^T > k^N}$. In the latter case, capital accumulation is solely determined by the *wealth effect* which lowers c^N as indicated by the second term on the right-hand side. Consequently, the investment flow turns immediately positive and capital accumulates until time $\mathcal{T}$. If $\underline{k^N > k^T}$, the *wealth effect* works against the *relative price effect*. Because the real exchange rate depreciates on impact, this adjustment has a depressive effect on Y^N and influences positively c^N. It can be shown analytically that the latter effect more than outweighs the *wealth effect*, and physical capital monotonically decreases over the unstable period 1.

The initial current account response relies upon the strength of a *relative price effect* falling now on tradables, the standard *wealth effect* and a *direct effect* due to temporarily higher government spending:

$$\left.\frac{\mathrm{d}ca(0)}{\mathrm{d}g^T}\right|_{temp} = \left(Y_p^T - c_p^T\right) \left.\frac{\mathrm{d}p(0)}{\mathrm{d}g^T}\right|_{temp} + \sigma_c \frac{\tilde{c}^T}{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp} - 1. \tag{22}$$

where $\left(Y_p^T - c_p^T\right) < 0$ under the mild assumption that c_p^T is positive.[14] If $\underline{k^N > k^T}$, the current account is initially affected by three effects. The *relative price effect* and the *wealth effect*, captured respectively by the first and the second term on the right-hand side of (22), improve the trade balance by stimulating the production of the traded sector Y^T and by lowering the consumption of tradables c^T. The strength of these two effects depend on the duration of the fiscal policy. Unlike, the third term on the right-hand side of (22) reflects the *direct effect* which worsens the net exports through the rise in public demand for the traded good. If the expansionary budget policy is implemented over a short period, say $\mathcal{T} < \hat{\mathcal{T}}$, the *direct effect* predominates and the small country decumulates traded bonds. Instead, if the spending shock is long-living, the *relative price effect* and the *wealth effect* more than outweigh the direct effect since the initial real exchange rate depreciation is more pronounced and the marginal utility of wealth rises markedly. If $\underline{k^T > k^N}$, the current account deteriorates initially without ambiguity because the *relative price effect* is no longer effective.

The dynamic adjustment in the case $\underline{k^T > k^N}$ is illustrated in the left panel of figure 2(a). The small country starts from an initial equilibrium F_0 and converges towards the long-run equilibrium F_{temp}. Over period 1 during which the fiscal expansion is effective, the adjustment involves a positive investment and a current account deficit. The solution for net foreign assets displays an explosive component due to the smoothing behavior of households. While t approaches $\mathcal{T}$, the economy decumulates traded bonds at an increasing rate. When g^T is restored to its initial level, the system moves along a sustainable path, shown as line $N'N'$.

Figures 2(b) trace out the transitional paths of the capital stock and net foreign asset position when $\underline{k^N > k^T}$. The striking difference compared to the dynamic adjustment which arises after a permanent fiscal expansion is that the stock of foreign assets may exhibit a non-monotonic pattern if the public policy is implemented over a sufficiently long period, say $\mathcal{T} > \hat{\mathcal{T}}$ (see the right panel of figure 2(b)). Compared to a permanent fiscal shock, the transitional path followed by the current account is now influenced by a *smoothing effect* reflected by the first term on the

	$\mathrm{d}g^T$				$\mathrm{d}g^N$			
	Permanent		Temporary		Permanent		Temporary	
	$k^T > k^N$	$k^N > k^T$	$k^T > k^N$	$k^N > k^T$	$k^T > k^N$	$k^N > k^T$	$k^T > k^N$	$k^N > k^T$
Long-Term Effects								
K	+	-	+	-	-	+	+	-
n	-	+	-	-	+	-	-	-
a	0	+	-	-	0	-	-	-
c	-	-	-	-	-	-	-	-
Short-Term Effects								
$I(0)$	+	-	+	-	-	+	-	-/+
$ca(0)$	-	+	-	-/+	+	-	+	-
$S(0)$	0	+	0	-/+	0	-	0	-/+
$c(0)$	-	-	-	-	-	-	-	-
$p(0)$	0	-	0	-	0	+	0	+

Figure 1: Qualitative Effects of Permanent and Temporary Fiscal Policy

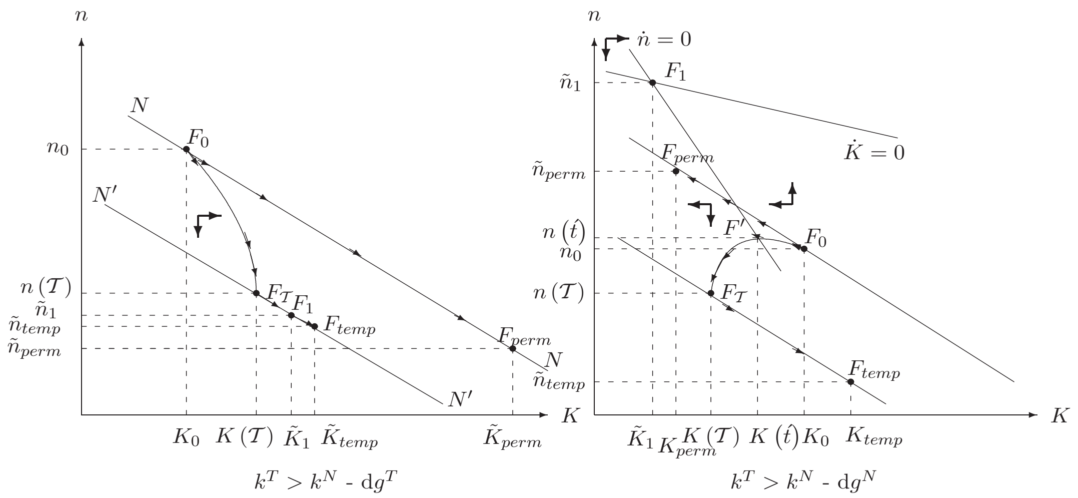
right-hand side besides the standard influence of capital accumulation:

$$ca(t) = \dot{n}(t) = -e^{-r^{\star}(\mathcal{T}-t)}\mathrm{d}g^T + \frac{N_1}{\mu_1 - r^{\star}}\dot{K}(t) \gtreqless 0. \tag{23}$$

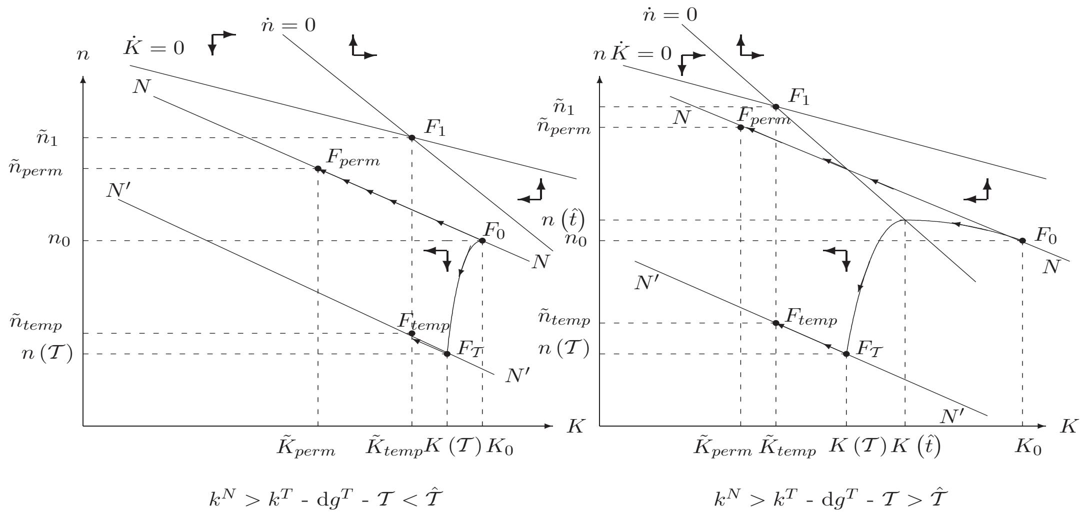
The explosive component reflects the fact that households understood perfectly that the spending shock is solely transitory such that they choose to lower their real consumption but by less than the fall of their disposable income. This behavior worsens the current account. The second term represents the positive influence on the current account induced by the decumulation in the physical capital stock. The longer-lasting the fiscal expansion, the more likely a current account improvement over a first phase followed by a current account deficit over a second phase due to the *smoothing effect*. The external asset position worsens until the fiscal policy is removed at time $\mathcal{T}$. Over the stable period, i. e. from $\mathcal{T}$ onwards, physical capital stock keeps on falling and the current account improves (along the line $N'N'$) until the small country reaches the long-run equilibrium at point F_{temp} in figure 2(b).

5 Permanent vs. Temporary Increase in Government Spending on the Non Traded Good

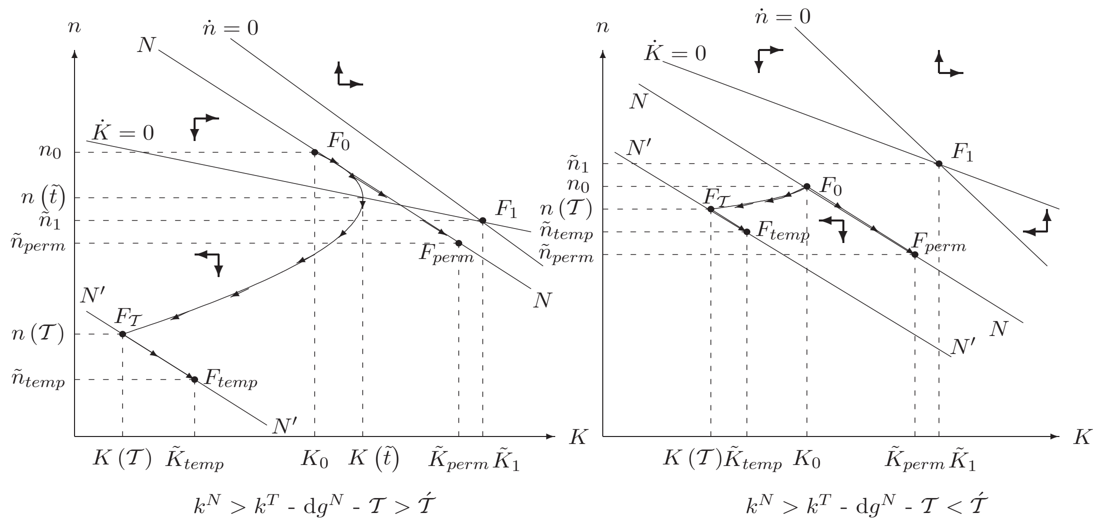
Until now, we have analyzed the effects of a fiscal expansion falling on g^T. Based on empirical evidence which points out that government expenditure tend to fall heavily on non traded goods, we analyze the effects of a temporary increase in g^N. Our main findings are as follows. First, different from a temporary rise in g^T, investment adjustment may display a non-monotonic pattern over $(0, \mathcal{T})$. Second, if the shock is not too long-living, investment is crowded-out by the transitory fiscal shock in the short-term, irrespective of the relative sectoral capital intensity. Third,



(a) Permanent vs. Temporary Increase in g^T and g^N - $k^T > k^N$



(b) Permanent vs. Temporary Increase in g^T - $k^N > k^T$



(c) Permanent vs. Temporary Increase in g^N - $k^N > k^T$

Figure 2: Fiscal Expansion in the (K, n)-space

we show formally that a permanent fiscal expansion is not an infinitely long-lived expansionary budget policy.

An Unanticipated Permanent Increase in g^N

Compared to a permanent increase in g^T, steady-state changes in the aggregate capital stock and the internationally traded bonds holding are reversed whenever public spending falls on g^N. If the non traded (traded) sector is relatively more capital intensive, $\tilde{K}$ will increase (decrease) in order to meet the higher demand for non traded goods. Because the capital stock shifts towards the more capital intensive sector, output of the traded sector unambiguously falls at the new steady-state state in either case. If $k^N > k^T$ ($k^T > k^N$), the trade balance improves (worsens) in the long-run. In the steady-state equilibrium, the current account must zero such that the small country holds a lower (higher) stock of internationally traded bonds.

The transitional path in the case $\underline{k^N > k^T}$ is depicted in figure 2(c). The economy starts from an initial steady-state at point F_0. An unanticipated permanent rise in government spending on the non traded good raises the demand for non traded goods and leads to an initial appreciation in the real exchange rate. The initial rise in the relative price p to the level $p(0)_{perm}$ attracts resources in the non traded sector. Beside this *relative price effect*, the *wealth effect* exerts a negative influence on c^N. Because the strength of these two effects are sufficiently strong for more than outweighing the *direct effect* reflected by the rise in g^N, investment is crowded-in by the permanent fiscal expansion. While the small country converges towards its new long-run equilibrium at F_{perm}, the current account deteriorates monotonically.

Assuming that $\underline{k^T > k^N}$, the *direct effect* predominates and leads to a capital decumulation. As illustrated in the right panel of figure 2(a), the economy moves along the stable branch labelled NN from point F_0 to point F_{perm}. The aggregate capital stock declines and agents accumulate internationally traded bonds.

An Unanticipated Temporary Increase in g^N

We explore now the effects of an unexpected temporary expansionary budget policy falling on g^N. In the case $\underline{k^T > k^N}$, the marginal utility of wealth rises initially but its jumps is moderated compared to a permanent fiscal expansion:

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} = \lambda_{g^N}\left(1 - e^{-r^\star \mathcal{T}}\right) > 0. \tag{24}$$

Interestingly, the steady-state variations of $\tilde{K}$ and $\tilde{n}$ are reversed compared to an unanticipated rise in g^N. An important finding is that the capital stock reaches a higher level at the new long-term equilibrium; more specifically, the magnitude of the economic expansion depends upon the duration of the fiscal shock:

$$\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N}\right|_{perm} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{perm} + K_{g^N} < 0 < \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N}\right|_{temp} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp}. \tag{25}$$

While a permanent fiscal expansion depresses investment, a rise in government spending implemented solely temporarily has a positive impact on capital accumulation. Its magnitude rises with the length of implementation for two reasons: the *wealth effect* turns out to be larger and the economic policy is expected to be removed at some date (although in the distant future) such that the direct effect reflected by K_{g^N} vanishes.

We investigate now the impact effects, starting first by investment behavior:

$$\left.\frac{\mathrm{d}I(0)}{\mathrm{d}g^N}\right|_{temp} = \sigma_c \frac{\tilde{c}^N}{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} - 1 < 0. \tag{26}$$

The striking difference compared to a permanent fiscal expansion on g^N is that the initial upward jump in $\bar{\lambda}$ is now dampened and consequently the induced fall in c^N is moderated. The negative investment flow is thus much more pronounced in absolute terms and the capital stock decreases along an unstable path over the entire period 1 as it is illustrated in the right panel of figure 2(a). Keeping in mind that the marginal utility of wealth remains constant after its once-for-all change at time $t = 0^+$, once the government policy is removed, investment jumps upward for the market-clearing condition to hold, that is $\mathrm{d}I\left(\mathcal{T}^+\right) = -\mathrm{d}g^N\left(\mathcal{T}\right) > 0$. Over the stable period 2, capital accumulates until the small country reaches its long-run equilibrium at point F_{temp}.

The current account dynamics are a little more complex compared to a permanent fiscal expansion since a *smoothing effect* is now at work. Transitional dynamics can be disentangled in two phases by noting that there exists a date $t = \acute{t}$ (with $ca\left(\acute{t}\right) = 0$) such that the stock of foreign assets overshoots along the stable trajectory, as it is depicted in the right panel of figure 2(a). The net foreign asset position improves initially while the negative investment flow more than outweighs the *smoothing* effect:

$$\left.\frac{\mathrm{d}ca(0)}{\mathrm{d}g^N}\right|_{temp} = \tilde{p}\left(1-\alpha_c\right)\left(1 - e^{-r^\star\mathcal{T}}\right) > 0 \tag{27}$$

At time $\acute{t}$, these two effects cancel each other. Subsequently, the current account deteriorates because the smoothing behavior predominates, such that $ca(t) \leq 0$ over $t \geq \acute{t}$. Once the public policy has been removed, the positive investment flow deteriorates the current account. At the new steady-state, the capital stock is permanently increased and the net foreign asset position of the small country is definitively worsened.

In the case $\underline{k^N > k^T}$, transitional dynamics turn out to be more interesting in economic terms because a *real interest rate* is now at work. Whenever the non traded sector features a higher capital-labor ratio, the real exchange rate dynamics do no longer degenerate. The once-for-all jump in $\bar{\lambda}$ is now much more moderated thanks to a *real interest rate effect*:

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} = \lambda_{g^N} \left\{ \left(1 - e^{-\mu_2 \mathcal{T}}\right) - \frac{\mu_2 \left(e^{-r^\star \mathcal{T}} - e^{-\mu_2 \mathcal{T}}\right)}{\left[\mu_2 + \frac{r^\star}{\mu_2} \frac{\tilde{c}^N}{\tilde{p}} \sigma_c \omega_2^1\right]} \right\} > 0, \tag{28}$$

From expression (28), a transitory fiscal expansion exerts on $\bar{\lambda}$ two conflictory influences. The (positive) first term on the right-hand side of (28) indicates that the change in $\bar{\lambda}$ equals the change after a permanent rise in g^N, λ_{g^N}, scaled-down by the term $0 < \left(1 - e^{-\mu_2 \mathcal{T}}\right) < 1$. In addition, a change in government spending falling on g^N gives rise to a *real interest rate effect* reflected by the (negative) second term. Following an increase in g^N, the real exchange rate appreciates initially which in turns magnifies the standard *wealth effect* reducing real consumption. This behavior softens the decumulation of foreign bonds (as reflected by $e^{-r^\star \mathcal{T}}$) but plays negatively upon investment (as reflected by $-e^{-\mu_2 \mathcal{T}}$). Since the former influence prevails over the latter, the *real interest rate effect* dampens the standard *wealth effect*. It can be shown that $\bar{\lambda}$ unambiguously jumps upwards. We shall discuss long-run changes at the end of this section.

The initial investment flow is the result of three conflictory forces and its sign is not clear-cut:

$$\left.\frac{\mathrm{d}I(0)}{\mathrm{d}g^N}\right|_{temp} = \left(Y_p^N - c_p^N\right) \left.\frac{\mathrm{d}p(0)}{\mathrm{d}g^N}\right|_{temp} + \sigma_c \frac{\tilde{c}^N}{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} - 1, \tag{29}$$

Beside the *relative price effect* (first term on the right-hand side of (29)) and the *wealth effect* (second term) that influences positively capital accumulation, the *direct effect* (third term) reflects the crowding-out of investment expenditure in response to government spending. Compared to a permanent fiscal expansion, a rise in g^N may eventually crowd-out capital accumulation if the policy is implemented over a short period, say $\mathcal{T} < \acute{\mathcal{T}}$. By contrast, if the spending shock is persistent, investment rises initially. Nevertheless, there exists a critical value of time, $t = \tilde{t} > 0$, such that the stock of physical capital reaches a turning point during its transitional path, i. e. $I\left(\tilde{t}\right) = 0$, and investment will be crowded-out thereafter.

The initial current account response can be formally described as follows:

$$\left.\frac{\mathrm{d}ca(0)}{\mathrm{d}g^N}\right|_{temp} = \frac{N_1}{\mu_1 - r^\star} \left.\frac{\mathrm{d}I(0)}{\mathrm{d}g^N}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) - \tilde{p}\left(e^{-r^\star \mathcal{T}} - e^{-\mu_2 \mathcal{T}}\right) < 0. \tag{30}$$

The first term on the right-hand side of (30) indicates that the initial current account offsets investment, the latter being moderated compared to a permanent fiscal expansion by a scaled-down factor $0 < \left(1 - e^{-r^\star \mathcal{T}}\right) < 1$. In words, if the expansionary budget policy is long-lived, capital accumulation takes place which in turn deteriorates the foreign asset position. The second term on the right-hand side of (30) ($-\tilde{p}e^{-r^\star \mathcal{T}} < 0$) represents the standard *smoothing effect* which deteriorates the net foreign asset position due to a negative savings flow. The third term ($\tilde{p}e^{-\mu_2 \mathcal{T}} > 0$) reflects the *real interest effect* which plays negatively on capital accumulation and therefore influences positively the current account. It can be shown analytically that the current account unambiguously worsens at the time the public policy is implemented and international assets are decumulated monotonically over entire period 1.

Once the public policy is permanently removed at time $\mathcal{T}$, the small country moves along a stable branch denoted by $F_{\mathcal{T}}F_{temp}$ displayed in figure 2(c). Whatever the duration of the fiscal shock, the investment flow turns positive for the non traded market clearing condition to hold while g^N is restored to its initial level. Consequently, the net foreign asset position keeps on worsening until the small country reaches the long-run equilibrium F_{temp}. Henceforth, we may infer that the stock of financial wealth held by households is permanently lowered.

A Permanent Fiscal Policy is not an Infinitely-Lived Transitory Fiscal Policy

Long-term effects of an unanticipated temporary fiscal expansion merits some comments. When the model features the zero-root property, transitory expansionary budget policies have permanent effects. More importantly, long-run changes after government spending shocks may be reversed compared to a permanent public policy. In particular, making use of (17), we find that an unanticipated temporary fiscal expansion always worsens the net foreign asset position in the long-run:

$$\left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^j}\right|_{perm} = v_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^j}\right|_{perm} + v_{g^j} \gtrless 0, \qquad \left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^j}\right|_{temp} = v_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^j}\right|_{temp} < 0, \quad j = T, N, \tag{31}$$

with $v_{g^j} > 0$. From (31), after a permanent fiscal expansion, steady-state change in the stock of international assets is the sum of two influences: a *wealth effect* induced by the change in $\bar{\lambda}$ and a *direct effect* driven by permanently raised government spending. Differently, after a temporary fiscal expansion, the *direct effect* is ineffective since g^j is perfectly expected to revert back to its initial level. Because $\bar{\lambda}$ jumps upward in all cases, net exports improve which requires a fall in the stock of internationally traded bonds for the current account to be balanced in the long-run. The longer-lasting the fiscal expansion, the stronger the *wealth effect* and the greater the fall in $\tilde{n}$.

We turn to the long-run change in the capital stock after a fiscal expansion falling on g^N, considering the most interesting case, i. e. $\underline{k^N > k^T}$. This discussion allows to state formally that a permanent fiscal expansion is not an infinitely-lived transitory expansionary policy. To provide a formal proof, we differentiate function (17) and we let the parameter $\mathcal{T}$ tend to infinity which implies:

$$\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N}\right|_{temp}^{\mathcal{T}\to\infty} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp}^{\mathcal{T}\to\infty} = K_{\bar{\lambda}}\lambda_{g^N} < 0. \tag{32}$$

From (32), the capital stock unambiguously falls in the long-run following an infinitely-lived transitory expansionary policy while after an unanticipated permanent rise in g^N, $\tilde{K}$ unambiguously rises:

$$\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N}\right|_{perm} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{perm} + K_{g^N} = \underbrace{K_{\bar{\lambda}}\lambda_{g^N}}_{(-)} + \underbrace{K_{g^N}}_{(+)} > 0. \tag{33}$$

This result can be explained in a simple way. After an infinitely-lived transitory shock, the *wealth effect* extends over an infinite period like after a permanent shock. Whenever a fiscal expansion falling on g^N is implemented transitorily, government spending returns at time $\mathcal{T}$ to its original level, although in a very distant future. This discrete fall in g^N implies a lower demand of non tradables which in turn requires a decrease in supply and thus in the capital stock. While after a permanent fiscal expansion, the *direct effect* crowds-in investment in the long-run, its absence always crowds-out capital accumulation whenever government purchases are raised solely temporarily. The longer-lasting the fiscal expansion, the stronger the *wealth effect* and the larger the fall in $\tilde{K}$.

6 The Quantitative Role of Duration of Fiscal Expansion

The findings of the previous section point out the importance of differentiating between permanent and temporary fiscal shocks on the one hand, and long-lived and short-lived expansionary budget policies on the other. In this section, we compare quantitatively the effects of permanent and temporary shifts in government expenditure. To conduct the numerical simulations we assume that the instantaneous utility function is of the CRRA form:

$$u(c) = \frac{1}{1-\frac{1}{\sigma_c}} c^{1-\frac{1}{\sigma_c}}. \tag{34}$$

The traded and the non traded good are aggregated by the means of a CES function:

$$c\left(c^T, c^N\right) = \left[\varphi^{\frac{1}{\phi}} \left(c^T\right)^{\frac{\phi-1}{\phi}} + (1-\varphi)^{\frac{1}{\phi}} \left(c^N\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \tag{35}$$

with φ the weight attached to the traded good in the overall consumption bundle ($0 < \varphi < 1$) and ϕ the intratemporal elasticity of substitution ($\phi > 0$). The production functions are supposed to take a Cobb-Douglas form:

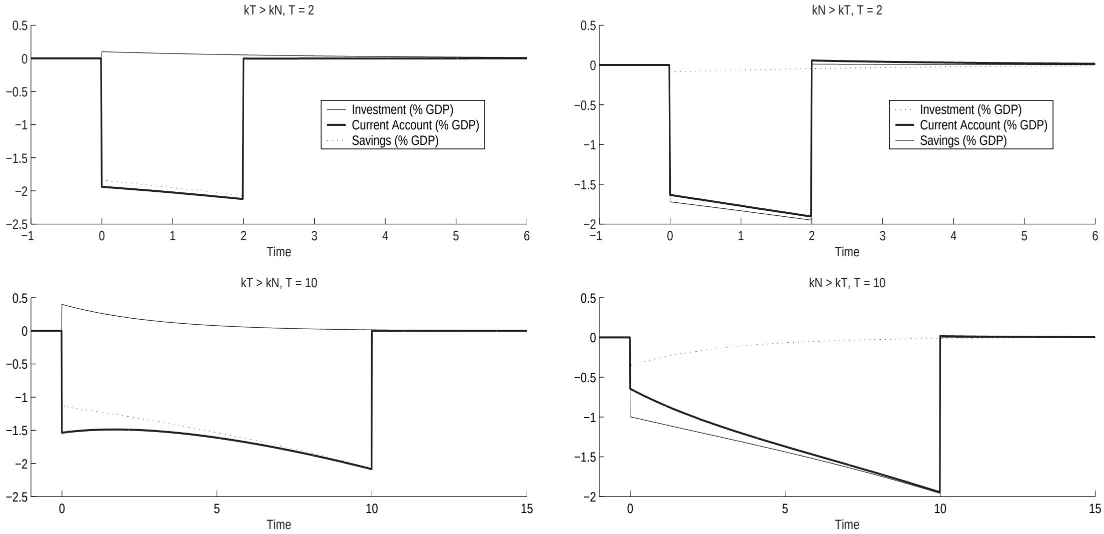
$$f\left(k^T\right) = \left(k^T\right)^{\theta}, \quad 0 < \theta < 1, \qquad h\left(k^N\right) = \left(k^N\right)^{\beta}, \quad 0 < \beta < 1, \tag{36}$$

where the parameters θ and β indicate the degree of capital intensity in the traded and non traded sector.

The values of parameters are set to be consistent with standard estimates. The intertemporal elasticity of substitution denoted by σ_c is set to 0.7, a value falling in the range 0.5 and 1 of empirical estimates (see e. g. Cashin and McDermott [2003]).[15] The intratemporal elasticity of substitution is set to 2 implying that traded and non traded goods are substitutes in consumption (i. e. $c_p^T > 0$). Following Stockman and Tesar [1995], the parameter φ is assigned such that the share of non tradables in total expenditure denoted by α_c falls in the range 40-60%. At this end, φ is set to 0.5. Regarding the production-side, if the traded sector is more capital intensive (i. e. $k^T > k^N$), parameters θ and β take the values 0.45 and 0.35. With the reversal capital intensity, we impose that $\theta = 0.35$ and $\beta = 0.45$. These values are close to sectoral labor intensities estimated by Kakkar [2003].[16] The world interest rate is chosen to be 0.06. The benchmark values for g^T and g^N are set to get consistent government expenditure-GDP ratios (g^T/Y^T, g^N/Y^N and g/Y).[17] Tables 5(a) and 5(b) report the steady-state values of key economic variables.[18]

We now investigate the effects of permanent and temporary shifts in government purchases which last for $\mathcal{T}$ years in the latter case. Fiscal expansions are calibrated in order to simulate an increase in the ratio government spending-GDP (g/Y) by 2 percentage points of GDP. Table 6 displays the short- and long-term responses of key economic variables following permanent and temporary increase in g^T and g^N; we consider a two-year, five-year and a ten-year fiscal shock. First, as we underlined it previously, fiscal shocks drive down the stock of international assets in the long-run in all cases. This finding is supported by recent empirical evidence provided by Bussière et al. [2007] who find that for the G-7 countries, an increase in government budget deficit by 1 percentage point of GDP will on average worsen the current account by 0.14 percentage points of GDP. Second, our quantitative analysis shows that, irrespective of capital intensities and the good on which public spending falls, temporary government expenditure shifts have greater long-term impacts on the net foreign asset position. The explanation is straightforward. While after an unexpected permanent fiscal shock, the steady-state change in internationally traded bonds holding is driven by a *direct effect* (induced by permanently raised government expenditure) working against a *wealth effect*, after a temporary fiscal shock the latter effect solely is at stake. Because the rise in the marginal utility of wealth improves net exports, the net foreign asset position must worsen markedly for the current account equilibrium to hold in the long-run.[19] Our third result relates to capital accumulation. Empirical studies, in particular Perotti [2005] and Bussière et al. [2007] find a significant negative effect of fiscal shocks on investment in OECD countries. Responses of investment displayed in Table 6 highlights this empirical evidence. Compared to a permanent expansionary budget policy, it is striking to state that a temporary fiscal shock crowds-out investment in the short-run in most cases, in particular when the government spending shock falls on g^N.

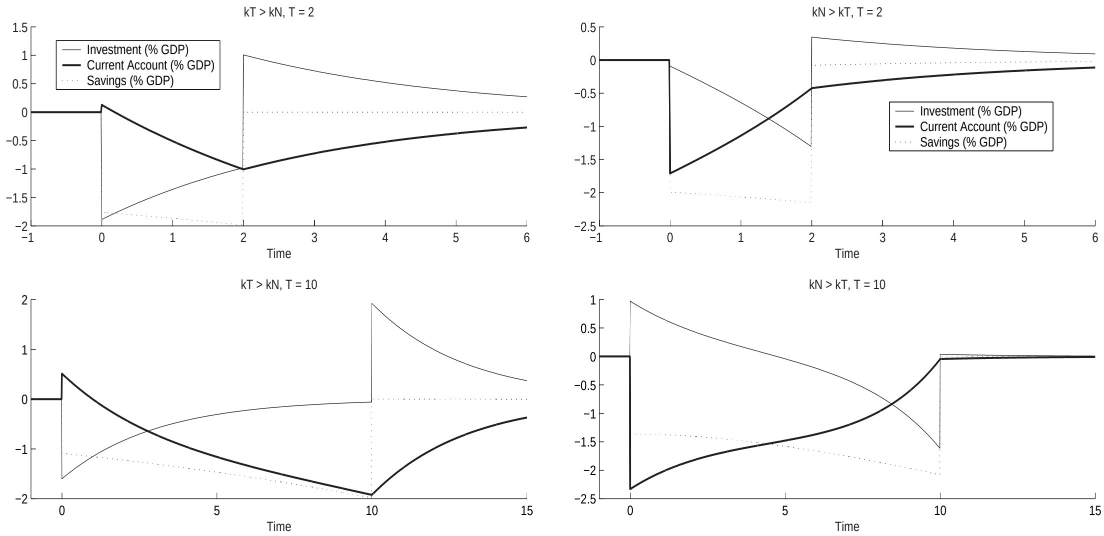
Figures 3 trace out the computed transitional paths for investment, savings and current account after an unanticipated temporary rise in g^T. Our two main results are as follows. Irrespective of the sectoral capital intensities, the shorter-living the fiscal expansion, the stronger the *smoothing effect* and the larger the initial current account deficit. This result is in line with empirical evidence provided by Freund [2005] and supports the view that current account deficits are mostly demand driven. In the case $k^T > k^N$, the temporary nature of the fiscal policy restores the transitional dynamics for savings due to the smoothing behavior which mostly explains the current deficit. In the case $k^N > k^T$, while after a permanent rise in g^T, the small country reaches its new long-run equilibrium with a higher stock of foreign assets, a temporary fiscal policy worsens the net foreign asset position both in the short- and the long-run.[20] Regarding



(a) $k^T > k^N$

(b) $k^N > k^T$

Figure 3: Transitional Paths for Unexpected Permanent and Temporary Increase in g^T



(a) $k^T > k^N$

(b) $k^N > k^T$

Figure 4: Transitional Paths for Unexpected Permanent and Temporary Increase in g^N

investment behavior, its initial response raises with the length of the lead time $\mathcal{T}$. If the fiscal shock is long-living, consumption of non tradables falls markedly because of the *wealth effect* which in turn leads to a positive or a negative investment flow depending on wether $k^T \gtrless k^N$.

The computed transitional paths following an unanticipated temporary rise in g^N are depicted in figures 4. As shown in section 5 in the case $k^N > k^T$, investment may now exhibit a non-monotonic pattern during the period over which the public policy is implemented (see figures 4(b)). If the fiscal expansion is short-living, investment is crowded-out on impact since the *direct effect* more than outweighs the *relative price effect* driven by the initial real exchange rate appreciation coupled to the standard *wealth effect*. By contrast, if the fiscal policy is a ten-year shock, capital accumulates over five years; then, along the trajectory, investment flow turns negative until the policy is removed at time $\mathcal{T} = 10$. The transitional dynamics for investment over unstable period 1 can be split into two phases. Over a first phase, the trajectory displays a convex pattern as the term (of its solution) associated to the stable eigenvalue (μ_1) predominates. Over a second phase, the term associated to the unstable root predominates such that investment turns negative. The capital stock moves along a decreasing trajectory with a slope becoming steeper in absolute terms as time t approaches the removal date, say $\mathcal{T} = 10$. Irrespective of the sectoral capital intensity, investment is crowded-in when the public spending is restored back to its initial level. In the case $k^N > k^T$, decumulation of the capital stock is slightly softened by accumulation after time $\mathcal{T}$ such that $\tilde{K}$ unambiguously falls in the long-run while a permanent expansionary budget policy crowds-in capital accumulation.

7 Conclusion

The extension of the two-sector model by Turnovsky and Sen [1995] to the study of temporary fiscal expansions turns out to be particularly interesting because we show formally that a transitory rise in government expenditure may induce opposite responses of macroeconomic aggregates compared to a permanent policy both in the short-run and in the long-run. More specifically, the existence of the zero-root property implies that a temporary public policy is not a dampening down of the permanent effect of the corresponding permanent policy. In words, a permanent fiscal expansion is not an infinitely-lived temporary expansionary budget policy, the latter having permanent effects.

What are the main economic policy lessons drawn from the a two-sector small open economy model inhabiting by infinitely-living agents, having time separable preferences and facing perfect international capital markets? The zero-root property requires to differentiate carefully between permanent an temporary fiscal policies on the one hand and between short-lived and long-lived transitory fiscal shocks on the other. One important finding is that, irrespective of the good on which the rise in government spending falls and irrespective of the duration of the public policy, whenever the non traded sector features a higher capital-labor ratio, the stock of physical capital and international assets holding are permanently reduced in the long-run. This result must signal to decisions-makers that an expansionary budget policy may eventually crowd-in investment in the short-run (under some conditions), but this expansionary effect will last a short period and investment will be crowded-out in the long-run. By contrast, irrespective of the good on which the rise in government spending falls, when the traded sector is relatively more capital intensive, a temporary rise in public expenditure always raises the capital stock in the long-run although the net foreign asset asset position worsens in all cases. To summarize, short-term and and long-term effects after a temporary rise in government spending may be reversed compared to those obtained after a permanent public policy.

Our contribution could be usefully extended by introducing a labor-leisure trade-off. With elastic labor, the effects of both permanent and temporary shocks in public consumption will change substantially. In particular, the capital stock will be positively correlated with the marginal utility of wealth if the elasticity of labor supply is not too small. This implies that both permanent and temporary fiscal shocks will crowd-in investment in the long-run, irrespective of the good on which government spending falls and the duration of the expansionary budget policy. This extension comes at a price since the transitional paths must be computed numerically.

Notes

[1]According to Cashin and McDermott [2003], the non traded share in GDP represents more than 60% of GDP for a group of OECD countries.

[2]Using disaggregated data from 30 countries Morshed an Turnovsky [2004] estimate that non traded government spending represent at least 83 % of total public spending.

[3]Cashin and McDermott [2003] provide some empirical evidence regarding the importance of the intratemporal substitution of consumption between traded and non traded goods on private savings.

[4]In a small open economy model that consists of a large number of identical and infinitely living agents with perfect foresight, the equality between the constant domestic rate of time preference and the exogenous world interest rate must be imposed to insure a finite interior steady-state value for the marginal utility of wealth which gives rise to the zero-root property.

[5]Most key results of the paper can be retrieved in a Technical Appendix available from authors on request.

[6]The unit cost dual function, $p_c(.)$, is defined as the minimum total expense in consumption goods, E, such that $c = c\left(c^T(t), c^N(t)\right) = 1$, for a given level of real exchange rate, p. The minimized unit cost function depends on the real exchange rate and is expressed in terms of the traded good. The consumption price index has the following properties: $p_c(p) > 0$, $p_c'(p) > 0$, $p_c''(p) < 0$.

[7]We abstract from capital depreciation for simplicity.

[8]Substituting (11) into the production functions, we can solve for the traded and the non traded output: $Y^T = Y^T(K, p)$ and $Y^N = Y^N(K, p)$ with $Y_p^T < 0$ and $Y_p^N > 0$.

[9]General solutions for K and p are given by:

$$K(t) - \tilde{K} = B_1 e^{\mu_1 t} + B_2 e^{\mu_2 t}, \quad p(t) - \tilde{p} = \omega_2^1 B_1 e^{\mu_1 t} + \omega_2^2 B_2 e^{\mu_2 t}, \tag{37}$$

where B_1 and B_2 are constants to be determined and ω_2^i is the eigenvector associated with the eigenvalue μ_i (with $i = 1, 2$). We normalized the eigenvector ω_1^i to unity.

[10]Dynamic equation for real consumption is obtained by differentiating (7a) with respect to time and making use of (7f):

$$\dot{c} = \sigma_c c \left(r^\star - \alpha_c \frac{\dot{p}}{p} - \delta\right). \tag{38}$$

[11]A transitorily higher relative price of non tradables calls for a real exchange rate depreciation which raises the consumption-based real interest rate and thus lowers real consumption. If the intertemporal elasticity of substitution is low, say less than one as empirical evidence suggest, this effect is more than offset by the other effects at work described in the text.

[12]The steady-state value of the real exchange rate remains constrained by the equality between the capital marginal product in the non traded sector and the fixed world interest rate such that $\tilde{p}$ does not depend on $\bar{\lambda}$ or public policy parameters.

[13]This can be seen more formally by differentiating the short-run static solution for consumption with respect to g^T:

$$\left.\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^T}\right|_{perm} < \left.\frac{\mathrm{d}c(0)}{\mathrm{d}g^T}\right|_{perm} = c_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{perm} + c_p \left.\frac{\mathrm{d}p(0)}{\mathrm{d}g^T}\right|_{perm} < 0. \tag{39}$$

[14]The elasticity of c^T with respect to p is given by $\frac{c_p^T p}{c^T} = \alpha_c(\phi - \sigma_c)$. Its sign is ambiguous and relies upon the sizes of the intratemporal elasticity of substitution ϕ between tradables and non tradables and the intertemporal elasticity substitution σ_c. The empirical literature finds that a ϕ close to 1.5-2 and σ_c less than one. Consequently, a positive elasticity is in line with empirical estimates.

[15]Benchmark values for the intertemporal and the intratemporal elasticities of substitution come from estimates of Cashin and McDermott [2003].

[16]Kakkar [2003] provides labor sectoral intensities for a panel of fourteen OECD countries. His estimates suggest

that seven countries possess a traded sector which is more capital intensive with $\theta \in [0.39; 0.54]$ and $\beta \in [0.34; 0.51]$. In the case $k^T > k^N$, capital intensities are in average equal to: $\hat{\theta} = 0.46$ and $\hat{\beta} = 0.39$. If the non traded sector is more capital intensive, the parameters θ and β belong to the respective range $[0.28; 0.48]$ and $[0.36; 0.59]$; in this case, average values are: $\hat{\theta} = 0.40$ and $\hat{\beta} = 0.47$.

[17]We denoted by g and Y the total public expenditure and the overall GDP which are defined as follows: $g = g^T + pg^N$ and $Y = Y^T + pY^N$.

[18]Transitional paths in figures 3-4 and values in table 6 for investment are expressed in terms of the traded good such that all variables are measured in terms of the numeraire.

[19]In particular, we can note that the fall in the stock of international assets is large given the shortness in the length of the rise in g/Y; for example, in the case $k^T > k^N$, $\tilde{n}$ falls by more 16% after two years following a transitory increase in g^T.

[20]In section 4, we show that the initial response of the current account is not clear-cut and depends on the length of the lead time $\mathcal{T}$. Figures 3(a) show that the current account deteriorates on impact, even if the fiscal expansion is strongly persistent. Even if we considered a long-lived fiscal shock, say a ten-year shock, the initial current account over GDP enters in deficit (-0.64%). Our simulations indicate that the fiscal shock must last at least 18.6 years to give rise to a current account surplus. In this case, despite the fact that the small country accumulates traded bonds initially, the external position turns negative just after four months.

8 Tables

	Steady-State Values	
	$\mathbf{k^T > k^N}$	$\mathbf{k^N > k^T}$
$\tilde{k}^T$	22.91	25.66
$\tilde{k}^N$	15.08	39.00
$\tilde{p}$	1.34	0.71
$\tilde{K}$	18.21	32.71
$\tilde{n}$	0.92	-10.40
$\bar{\lambda}$	0.24	0.38
$\tilde{c}$	2.47	2.25
α_c	0.43	0.59

(a) Steady-State Values

	Model		Data
	$\mathbf{k^T > k^N}$	$\mathbf{k^N > k^T}$	
g^T/Y^T	0.04	0.05	0.04
g^N/Y^N	0.42	0.44	0.41
g^N/g	0.93	0.92	0.93
g/Y	0.25	0.27	0.26
Y^N/Y	0.56	0.57	0.59

(b) Steady-State Ratios

Figure 5: Long-Run Levels of Key Economic Variables

	$\mathrm{d}\mathbf{g^T}$				$\mathrm{d}\mathbf{g^N}$			
	$k^T > k^N$				$k^T > k^N$			
	Permanent	Temporary			Permanent	Temporary		
		$\mathcal{T}=2$	$\mathcal{T}=5$	$\mathcal{T}=10$		$\mathcal{T}=2$	$\mathcal{T}=5$	$\mathcal{T}=10$
Long-Term								
$\mathrm{d}\tilde{K}$	0.41	0.05	0.11	0.18	-0.52	0.04	0.10	0.18
$\mathrm{d}\tilde{n}$	-10.80	-16.94	-38.82	-67.59	13.85	-16.21	-37.15	-64.68
$\mathrm{d}\tilde{a}$	0.00	-1.45	-3.33	-5.79	0.00	-1.39	-3.18	-5.54
Short-Term								
$\mathrm{d}I(0)/\tilde{Y}_0$	0.89	0.10	0.23	0.40	-1.14	-1.89	-1.76	-1.60
$\mathrm{d}ca(0)/\tilde{Y}_0$	-0.89	-1.94	-1.77	-1.54	1.14	0.13	0.29	0.51
$\mathrm{d}S(0)/\tilde{Y}_0$	0.00	-1.84	-1.54	-1.14	0.00	-1.76	-1.47	-1.09
	$k^N > k^T$				$k^N > k^T$			
	Permanent	Temporary			Permanent	Temporary		
		$\mathcal{T}=2$	$\mathcal{T}=5$	$\mathcal{T}=10$		$\mathcal{T}=2$	$\mathcal{T}=5$	$\mathcal{T}=10$
Long-Term								
$\mathrm{d}\tilde{K}$	-0.43	-0.05	-0.11	-0.20	0.32	-0.05	-0.12	-0.21
$\mathrm{d}\tilde{n}$	0.95	-1.11	-2.53	-4.41	-0.71	-1.15	-2.65	-4.63
$\mathrm{d}\tilde{a}$	0.11	-0.72	-1.64	-2.86	-0.08	-0.75	-1.72	-3.01
Short-Term								
$\mathrm{d}I(0)/\tilde{Y}_0$	-0.78	-0.09	-0.20	-0.35	0.58	-0.28	0.70	1.16
$\mathrm{d}ca(0)/\tilde{Y}_0$	0.96	-1.63	-1.21	-0.65	-0.71	-1.71	-2.46	-2.33
$\mathrm{d}S(0)/\tilde{Y}_0$	0.18	-1.72	-1.41	-1.00	-0.13	-1.99	-1.76	-1.37

Figure 6: Quantitative Effects of Permanent and Temporary Fiscal Policy

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Mathematical Appendix

A Short-Run Static Solutions

A.1 Short-Run Static Solutions for Consumption-Side

In this section, we compute short-run static solutions for real consumption and labor supply by making use of the first-order condition (7a):

$$c = c\left(\bar{\lambda}, p\right), \tag{40}$$

with

$$\begin{aligned} c_{\bar{\lambda}} &= \frac{\partial c}{\partial \bar{\lambda}} = -\sigma_c \frac{c}{\bar{\lambda}} < 0, & \text{(41a)} \\ c_p &= \frac{\partial c}{\partial p} = -\alpha_c \sigma_c \frac{c}{p} < 0, & \text{(41b)} \end{aligned}$$

where $\sigma_c = -u_c/u_{cc}c > 0$ corresponds to the intertemporal elasticity of substitution.

Denoting by ϕ the intratemporal elasticity of substitution between the tradable and the non tradable good, we can solve for c^T and c^N:

$$c^T = c^T\left(\bar{\lambda}, p\right), \quad c^N = c^N\left(\bar{\lambda}, p\right), \tag{42}$$

with

$$\begin{aligned} c^T_{\bar{\lambda}} &= -\sigma_c \frac{c^T}{\bar{\lambda}} < 0, \qquad c^T_p = \alpha_c \frac{c^T}{p}\left(\phi - \sigma_c\right) \lesseqgtr 0, & \text{(43a)} \\ c^N_{\bar{\lambda}} &= -\sigma_c \frac{c^N}{\bar{\lambda}} < 0, \qquad c^N_p = -\frac{c^N}{p}\left[\left(1-\alpha_c\right)\phi + \alpha_c \sigma_c\right] < 0. & \text{(43b)} \end{aligned}$$

A.2 Short-Run Static Solutions for Production-Side

Capital-Labor Ratios

From static optimality conditions (7b) and (7c), we may express sector capital-labor ratios as functions of the real exchange rate:

$$k^T = k^T(p), \quad k^N = k^N(p), \tag{44}$$

with

$$\begin{aligned} k^T_p &= \frac{\partial k^T}{\partial p} = \frac{h}{f_{kk}\left(k^N - k^T\right)}, & \text{(45a)} \\ k^N_p &= \frac{\partial k^N}{\partial p} = \frac{f}{p^2 h_{kk}\left(k^N - k^T\right)}. & \text{(45b)} \end{aligned}$$

We summarize our results graphically. In figure 7, we have represented the locus of points such that $f_k = ph_k$, denoted by $PmK_T = PmK_N$ and the locus of points such that $f\left(k^T\right) - k^T f_k\left(k^T\right) = p\left[h\left(k^N\right) - k^N h_k\left(k^N\right)\right]$, denoted by $PmL_T = PmL_N$. In words, along these two curves, the equality of marginal products for capital and labor are insured. The locus $PmK_T = PmK_N$ has a slope given by:

$$\left.\frac{\mathrm{d}k^N}{\mathrm{d}k^T}\right|_{PmK_T=PmK_N} = \frac{f_{kk}}{ph_{kk}} > 0, \tag{46}$$

and the locus $PmL_T = PmL_N$ has a slope given by:

$$\left.\frac{\mathrm{d}k^N}{\mathrm{d}k^T}\right|_{PmL_T=PmL_N} = \frac{k^T f_{kk}}{pk^N h_{kk}} > 0. \tag{47}$$

Inspecting analytical expressions (46) and (47), we see that the locus $PmK_T = PmK_N$ is more or less steeper than the locus $PmL_T = PmL_N$ depending on wether $k^N \gtrless k^T$. In figure 7, we emphasize the impact of a real exchange rate appreciation on the sectoral capital-labor ratios. A rise in the relative price of non-tradable goods shifts the locus $PmK_T = PmK_N$ to the left and the locus $PmL_T = PmL_N$ to the right. In the left panel of figure 7, the equality of marginal products between the two sectors are insured for lower capital-labor ratios and in the right panel, for higher capital-labor ratios.

Using the fact that $w \equiv f\left(k^T\right) - k^T f_k\left(k^T\right)$ and $r^K \equiv f_k\left(k^T\right)$ and substituting $k^T = k^T(p)$, we can determine the change in the wage rate-rental rate ratio induced by a variation in the real exchange rate:

$$\frac{\mathrm{d}\left(w/r^K\right)}{\mathrm{d}p} = -\frac{f_{kk}k_p^T\left(r^K k^T + w\right)}{\left(r^K\right)^2} \lesseqgtr 0 \quad \text{depending on wether} \quad k^N \gtrless k^T. \tag{48}$$

A rise in the relative price of non traded goods causes a shifting of capital stock and labor from the traded to the non-traded sector. If $k^N > k^T$, capital increases in relative scarcity which in turn leads to a fall in the wage rate-rental rate ratio. If $k^T > k^N$, capital increases in relative abundance which raises the ratio w/r^K.

Labor

Substituting (44) into sectoral inputs allocation constraints (7d) and (7e), we can solve for traded and non-traded labor as follows:

$$L^T = L^T(K,p), \quad L^N = L^N(K,p), \tag{49}$$

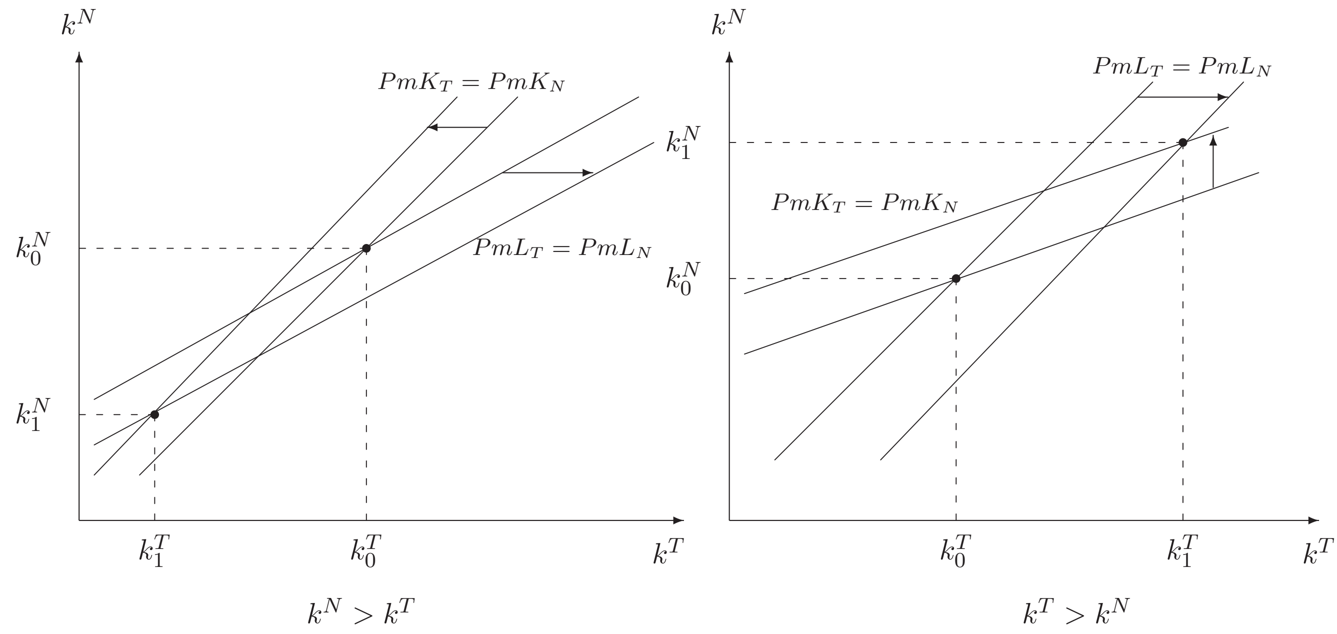

Figure 7: Sectoral capital-labor ratios and the real exchange rate

with

$$L_K^T = \frac{\partial L^T}{\partial K} = \frac{1}{k^T - k^N}, \tag{50a}$$

$$L_p^T = \frac{\partial L^T}{\partial p} = \frac{1}{(k^T - k^N)^2}\left[\frac{L^T h}{f_{kk}} + \frac{L^N f}{p^2 h_{kk}}\right] < 0, \tag{50b}$$

$$L_K^N = \frac{\partial L^N}{\partial K} = \frac{1}{k^N - k^T}, \tag{50c}$$

$$L_p^N = \frac{\partial L^N}{\partial p} = -\frac{1}{(k^T - k^N)^2}\left[\frac{L^T h}{f_{kk}} + \frac{L^N f}{p^2 h_{kk}}\right] > 0. \tag{50d}$$

From (50a) and (50c), when the capital stock rises, labor must shift to the sector which is relatively more capital intensive. If the relative of non traded goods increases, i .e. d$p > 0$, this causes a shift of labor from the traded to the non-traded sector, irrespective of the sectoral capital intensities.

Output

Substituting short-run static solutions for capital-labor ratios (44) and for labor (49) into the production functions, we can solve for the traded and non-traded output:

$$Y^T = Y^T(K, p), \quad Y^N = Y^N(K, p), \tag{51}$$

with

$$
\begin{aligned}
Y_K^T &= \frac{\partial Y^T}{\partial K} = \frac{f}{k^T - k^N}, && (52a)\\
Y_p^T &= \frac{\partial Y^T}{\partial p} = \frac{1}{(k^N - k^T)^2}\left[\frac{pL^T (h)^2}{f_{kk}} + \frac{L^N (f)^2}{p^2 h_{kk}}\right] < 0, && (52b)\\
Y_K^N &= \frac{\partial Y^N}{\partial K} = \frac{h}{k^N - k^T}, && (52c)\\
Y_p^N &= \frac{\partial Y^N}{\partial p} = -\frac{1}{p\,(k^T - k^N)^2}\left[\frac{pL^T (h)^2}{f_{kk}} + \frac{L^N (f)^2}{p^2 h_{kk}}\right] > 0. && (52d)
\end{aligned}
$$

A real exchange rate appreciation attracts resources from the traded to the non traded sector which in turn raises Y^N.

From equations (52a)-(52d), we deduce the following useful properties

$$
\begin{aligned}
Y_p^T + pY_p^N &= 0, && (53a)\\
Y_K^T + pY_K^N &= \frac{f - ph}{k^T - k^N} = f_k = ph_k > 0. && (53b)
\end{aligned}
$$

B Equilibrium Dynamics and Formal Solutions

Equilibrium Dynamics

Inserting short-run static solutions (10) and (11) into (7g) and (7h), linearizing these equations around the steady-state, and denoting $\tilde{x} = \tilde{K}, \tilde{p}$ the long-term values of $x = K, p$, we obtain in a matrix form:

$$
\left(\dot{K}, \dot{p}\right)^T = J\left(K(t) - \tilde{K}, p(t) - \tilde{p}\right)^T, \tag{54}
$$

where J is given by

$$
J \equiv \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}, \tag{55}
$$

with

$$
\begin{aligned}
b_{11} &= Y_K^N = \frac{\tilde{h}}{\tilde{k}^N - \tilde{k}_T} \gtrless 0, \quad b_{12} = Y_p^N - c_p^N > 0, && (56a)\\
b_{21} &= 0, \quad b_{22} = -\tilde{p} h_{kk} k_p^N = -\frac{\tilde{f}}{\tilde{p}\left(\tilde{k}^N - \tilde{k}^T\right)} \lesseqgtr 0. && (56b)
\end{aligned}
$$

By denoting μ the eigenvalue of matrix J, the characteristic equation for the matrix (55) of the linearized system can be written as follows

$$
\mu^2 - r^\star \mu - \tilde{p} h_{kk} k_p^N Y_K^N = 0, \tag{57}
$$

where we used the fact that $Y_K^N - \tilde{p} h_{kk} k_p^N = r^\star$ and $\delta = r^\star$. The determinant denoted by Det of the linearized 2×2 matrix (55) is unambiguously negative

$$\text{Det J} = b_{11} b_{22} = -\tilde{p} h_{kk} k_p^N Y_K^N = -\frac{\tilde{f}\tilde{h}}{\tilde{p}\left(\tilde{k}^N - \tilde{k}^T\right)^2} < 0, \tag{58}$$

and the trace denoted by Tr is positive

$$\text{Tr J} = b_{11} + b_{22} = r^\star > 0. \tag{59}$$

From (57), the characteristic root obtained from J writes as follows:

$$\mu_i \equiv \frac{1}{2}\left\{ r^\star \pm \sqrt{(r^\star)^2 + 4\frac{\tilde{f}\tilde{h}}{\tilde{p}\left(\tilde{k}^N - \tilde{k}^T\right)^2}} \right\} \gtrless 0, \quad i = 1, 2. \tag{60}$$

We denote by $\mu_1 < 0$ and $\mu_2 > 0$ the stable and unstable real eigenvalues, satisfying

$$\mu_1 < 0 < r^\star < \mu_2. \tag{61}$$

Since the system features one state variable, K, and one jump variable, p, the equilibrium yields a unique one-dimensional stable saddle-path.

Formal Solutions

General solutions paths are given by :

$$K(t) - \tilde{K} = B_1 e^{\mu_1 t} + B_2 e^{\mu_2 t}, \tag{62a}$$

$$p(t) - \tilde{p} = \omega_2^1 B_1 e^{\mu_1 t} + \omega_2^2 B_2 e^{\mu_2 t}, \tag{62b}$$

where we normalized ω_1^i to unity. The eigenvector ω_2^i associated with eigenvalue μ_i is given by

$$\omega_2^i = -\frac{\mu_i - b_{11}}{b_{12}}, \tag{63}$$

with

$$b_{11} = \tilde{h} L_K^N = \frac{\tilde{h}}{\tilde{k}^N - \tilde{k}_T} \gtrless 0, \tag{64a}$$

$$b_{12} = Y_p^N - c_p^N = Y_p^N - p_c'' \tilde{c} - p_c' c_p > 0. \tag{64b}$$

Case $k^N > k^T$

This assumption reflects the fact that the capital intensity of the non traded good sector exceeds the capital intensity of the traded sector. The stable and unstable eigenvalues given by

(60) may be rewritten in the following form

$$\mu_1 = -\frac{\tilde{f}}{\tilde{p}\left(\tilde{k}^N - \tilde{k}^T\right)} < 0, \tag{65a}$$

$$\mu_2 = \frac{\tilde{h}}{\left(\tilde{k}^N - \tilde{k}^T\right)} > 0. \tag{65b}$$

From (45) and (52), we may deduce the signs of several useful expressions

$$Y_K^N = \mu_2 > 0, \tag{66a}$$

$$Y_K^T = \tilde{p}\mu_1 < 0, \tag{66b}$$

$$\tilde{p}h_{kk}k_p^N = -\mu_1 > 0. \tag{66c}$$

We write out eigenvectors ω^i, corresponding to eigenvalue μ_i with $i = 1, 2$, to determine their signs

$$\omega^1 = \begin{pmatrix} 1 & (+) \\ \frac{\mu_1 - \mu_2}{\left(Y_p^N - c_p^N\right)} & (-) \end{pmatrix}, \quad \omega^2 = \begin{pmatrix} 1 & (+) \\ 0 & \end{pmatrix}. \tag{67}$$

Case $k^T > k^N$

This assumption reflects the fact that the capital intensity of the traded good sector exceeds the capital intensity of the non traded sector. The stable and unstable eigenvalues given by (60) may be rewritten in the following form

$$\mu_1 = \frac{\tilde{h}}{\left(\tilde{k}^N - \tilde{k}^T\right)} < 0, \tag{68a}$$

$$\mu_2 = -\frac{\tilde{f}}{\tilde{p}\left(\tilde{k}^N - \tilde{k}^T\right)} > 0, \tag{68b}$$

From (45) and (52), the following expressions can be derived

$$Y_K^N = \mu_1 < 0, \tag{69a}$$

$$Y_K^T = \tilde{p}\mu_2 > 0, \tag{69b}$$

$$\tilde{p}h_{kk}k_p^N = -\mu_2 < 0. \tag{69c}$$

We write out the two eigenvectors ω^i, corresponding to eigenvalues μ_i with $i = 1, 2$, to determine their signs

$$\omega^1 = \begin{pmatrix} 1 & (+) \\ 0 & \end{pmatrix}, \quad \omega^2 = \begin{pmatrix} 0 & \\ \frac{\mu_2 - \mu_1}{\left(Y_p^N - c_p^N\right)} & (+) \end{pmatrix}. \tag{70}$$

Formal Solution for n

We first linearize equation (7i) around the steady-state:

$$\dot{n}(t) = r^{\star}\left(n(t) - \tilde{n}\right) + Y_K^T\left(K(t) - \tilde{K}\right) + \left[Y_p^T - c_p^T\right]\left(p(t) - \tilde{p}\right). \tag{71}$$

where c_p^T is given by (43a). Inserting the stable solutions for $\left(K(t) - \tilde{K}\right)$ and $(p(t) - \tilde{p})$, the solution for the stock of net foreign assets writes as follows:

$$\dot{n}(t) = r^{\star}\left(n(t) - \tilde{n}\right) + Y_K^T\sum_{i=1}^{2} A_i e^{\mu_i t} + \left[Y_p^T - c_p^T\right]\sum_{i=1}^{2} A_i \omega_2^i e^{\mu_i t}. \tag{72}$$

Solving the differential equation leads to the following expression:

$$\begin{aligned} n(t) - \tilde{n} &= \left[(n_0 - \tilde{n}) - \frac{N_1 B_1}{\mu_1 - r^{\star}} - \frac{N_2 B_2}{\mu_2 - r^{\star}}\right] e^{r^{\star} t} \\ &+ \frac{N_1 B_1}{\mu_1 - r^{\star}} e^{\mu_1 t} + \frac{N_2 B_2}{\mu_2 - r^{\star}} e^{\mu_2 t}, \end{aligned} \tag{73}$$

with

$$N_i = Y_K^T + \left[Y_p^T - c_p^T\right]\omega_2^i, \quad i = 1, 2. \tag{74}$$

Invoking the transversality condition for intertemporal solvency, i. e. equation (8), the terms in brackets of equation (73) must be null and we must set $B_2 = 0$. Inserting the value for the constant B_1 given by equation (62a) evaluated at time $t = 0$, we obtain the linearized version of the nation's intertemporal budget constraint:

$$n_0 - \tilde{n} = \frac{N_1}{\mu_1 - r^{\star}}\left(K_0 - \tilde{K}\right). \tag{75}$$

The stable solution for net foreign assets finally reduces to:

$$n(t) - \tilde{n} = \frac{N_1 B_1}{\mu_1 - r^{\star}} e^{\mu_1 t}. \tag{76}$$

Case $k^N > k^T$

$$\begin{aligned} N_1 &= Y_K^T + \left[Y_p^T - c_p^T\right]\omega_2^1, \\ &= \tilde{p}\mu_2\left(1 + \frac{1}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c \omega_2^1\right) \gtrless 0, \end{aligned} \tag{77a}$$

$$\begin{aligned} N_2 &= Y_K^T + \left[Y_p^T - c_p^T\right]\omega_2^2, \\ &= Y_K^T = \tilde{p}\mu_1 < 0, \end{aligned} \tag{77b}$$

where we used the fact that $\omega_2^2 = 0$. The sign of $N_1/(\mu_1 - r^\star)$ is ambiguous and reflects the impact of the capital accumulation on the net foreign assets accumulation along a stable transitional path :

$$\dot{n}(t) = \frac{N_1}{\mu_1 - r^\star}\dot{K}(t).$$

where $\dot{K}(t) = \mu_1 B_1 e^{\mu_1 t}$. Since empirical studies suggest that the two macroeconomic aggregates are negatively correlated (see e. g. Glick and Rogoff [1995]), we will impose thereafter that

Assumption 1 $\frac{N_1}{\mu_1 - r^\star} < 0$ *which implies that* $N_1 > 0$.

The assumption under which $N_1 > 0$ may be rewritten as follows

$$\mu_2 > -\frac{\tilde{c}^N}{\tilde{p}}\sigma_c \omega_2^1. \tag{78}$$

Case $k^T > k^N$

$$\begin{aligned} N_1 &= Y_K^T + \left[Y_p^T - c_p^T\right]\omega_2^1, \\ &= Y_K^T = \tilde{p}\mu_2 > 0, \end{aligned} \tag{79a}$$

$$\begin{aligned} N_2 &= Y_K^T + \left[Y_p^T - c_p^T\right]\omega_2^2, \\ &= \tilde{p}\mu_1\left(1 + \frac{1}{\mu_1}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right) \gtrless 0, \end{aligned} \tag{79b}$$

where $Y_K^T = \tilde{p}\mu_2$ and $\omega_2^1 = 0$.

C Long-Run Effects of Unanticipated Permanent Fiscal Expansions

We totally differentiate the system (16) evaluated at the steady-state. In a matrix form, we get:

$$\begin{pmatrix} h_{kk}k_p^N & 0 & 0 & 0 \\ \left(Y_p^N - c_p^N\right) & Y_K^N & 0 & -p_c' c_{\bar{\lambda}} \\ \left(Y_p^T - c_p^T\right) & Y_K^T & r^\star & -(1-\alpha_c)\,p_c c_{\bar{\lambda}} \\ 0 & -\frac{N_1}{\mu_1 - r^\star} & 1 & 0 \end{pmatrix} \begin{pmatrix} \mathrm{d}\tilde{p} \\ \mathrm{d}\tilde{K} \\ \mathrm{d}\tilde{n} \\ \mathrm{d}\bar{\lambda} \end{pmatrix} = \begin{pmatrix} 0 \\ \mathrm{d}g^N \\ \mathrm{d}g^T \\ 0 \end{pmatrix} \tag{80}$$

Determinant of matrix of coefficients is given by

$$D = h_{kk}k_p^N \frac{p_c \tilde{c}\sigma_c}{\bar{\lambda}}\left[Y_K^N - r^\star \frac{\alpha_c}{\tilde{p}}\left(\tilde{p} + \frac{N_1}{\mu_1 - r^\star}\right)\right]. \tag{81}$$

We have to consider two cases, depending on wether the non-traded sector is more or less capital intensive than the traded sector :

$$\begin{aligned} D &= -\frac{\mu_1\mu_2 p_c\tilde{c}\sigma_c}{\tilde{p}\bar{\lambda}} > 0, \text{ case } k^T > k^N, & (82a)\\ D &= -\frac{\mu_1 p_c\tilde{c}\sigma_c}{\tilde{p}\bar{\lambda}}\left[\mu_2 + \alpha_c\frac{r^\star}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right] > 0, \text{ case } k^N > k^T. & (82b)\end{aligned}$$

The term in square brackets on the right-hand side of (82b) is positive if the following inequality holds

$$\mu_2 > -\alpha_c\frac{r^\star}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1. \tag{83}$$

From (78), this inequality is satisfied since $\alpha_c\frac{r^\star}{\mu_2} < 1$.

Long-Run Effects of an Unanticipated Permanent Shock in the Government Expenditure on the Traded Good

case $k^N > k^T$

$$\begin{aligned} \frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^T} &= -\frac{1}{\tilde{p}_c\left[1+\alpha_c\frac{r^\star}{(\mu_2)^2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]} < 0, & (84a)\\ \frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T} &= \frac{\alpha_c\bar{\lambda}}{\sigma_c\tilde{p}\tilde{c}^N\left[1+\alpha_c\frac{r^\star}{(\mu_2)^2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]} > 0, & (84b)\\ \frac{\mathrm{d}\tilde{p}}{\mathrm{d}g^T} &= 0, & (84c)\\ \frac{\mathrm{d}\tilde{L^T}}{\mathrm{d}g^T} &= \frac{\alpha_c}{\tilde{p}\tilde{h}\left[1+\alpha_c\frac{r^\star}{(\mu_2)^2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]} > 0, & (84d)\\ \frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T} &= -\frac{\alpha_c}{\tilde{p}\mu_2\left[1+\alpha_c\frac{r^\star}{(\mu_2)^2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]} < 0, & (84e)\\ \frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^T} &= \frac{\alpha_c}{\mu_2}\frac{\left[1+\frac{1}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]}{\left[1+\alpha_c\frac{r^\star}{(\mu_2)^2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]} > 0. & (84f)\end{aligned}$$

case $k^T > k^N$

$$
\begin{aligned}
\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^T} &= -\frac{1}{\tilde{p}_c} < 0, &(85a)\\
\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T} &= \frac{\alpha_c \bar{\lambda}}{\sigma_c \tilde{p} \tilde{c}^N} > 0, &(85b)\\
\frac{\mathrm{d}\tilde{p}}{\mathrm{d}g^T} &= 0, &(85c)\\
\frac{\mathrm{d}\tilde{L^T}}{\mathrm{d}g^T} &= \frac{\alpha_c}{\tilde{p}\tilde{h}} > 0, &(85d)\\
\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T} &= -\frac{\alpha_c}{\tilde{p}\mu_1} > 0, &(85e)\\
\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^T} &= \frac{\alpha_c}{\mu_1} < 0. &(85f)
\end{aligned}
$$

Long-Run Effects of an Unanticipated Permanent Shock in the Government Expenditure on the Non Traded Good

case $k^N > k^T$

$$
\begin{aligned}
\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^N} &= -\frac{\tilde{p}}{\tilde{p}_c} \frac{\left[1 + \frac{r^\star}{(\mu_2)^2} \frac{\tilde{c}^N}{\tilde{p}} \sigma_c \omega_2^1\right]}{\left[1 + \alpha_c \frac{r^\star}{(\mu_2)^2} \frac{\tilde{c}^N}{\tilde{p}} \sigma_c \omega_2^1\right]} < 0, &(86a)\\
\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N} &= \frac{\alpha_c \bar{\lambda}}{\sigma_c \tilde{c}^N} \frac{\left[1 + \frac{r^\star}{(\mu_2)^2} \frac{\tilde{c}^N}{\tilde{p}} \sigma_c \omega_2^1\right]}{\left[1 + \alpha_c \frac{r^\star}{(\mu_2)^2} \frac{\tilde{c}^N}{\tilde{p}} \sigma_c \omega_2^1\right]} > 0, &(86b)\\
\frac{\mathrm{d}\tilde{p}}{\mathrm{d}g^N} &= 0, &(86c)\\
\frac{\mathrm{d}\tilde{L^T}}{\mathrm{d}g^N} &= -\frac{(1-\alpha_c)}{\tilde{h}\left[1 + \alpha_c \frac{r^\star}{(\mu_2)^2} \frac{\tilde{c}^N}{\tilde{p}} \sigma_c \omega_2^1\right]} < 0, &(86d)\\
\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N} &= \frac{(1-\alpha_c)}{\mu_2\left[1 + \alpha_c \frac{r^\star}{(\mu_2)^2} \frac{\tilde{c}^N}{\tilde{p}} \sigma_c \omega_2^1\right]} > 0, &(86e)\\
\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^N} &= -\frac{\tilde{p}(1-\alpha_c)\left[1 + \frac{1}{\mu_2} \frac{\tilde{c}^N}{\tilde{p}} \sigma_c \omega_2^1\right]}{\mu_2\left[1 + \alpha_c \frac{r^\star}{(\mu_2)^2} \frac{\tilde{c}^N}{\tilde{p}} \sigma_c \omega_2^1\right]} < 0. &(86f)
\end{aligned}
$$

case $k^T > k^N$

$$\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^N} = -\frac{\tilde{p}}{\tilde{p}_c} < 0, \tag{87a}$$
$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N} = \frac{\alpha_c \bar{\lambda}}{\sigma_c \tilde{p}\tilde{c}^N} > 0, \tag{87b}$$
$$\frac{\mathrm{d}\tilde{p}}{\mathrm{d}g^N} = 0, \tag{87c}$$
$$\frac{\mathrm{d}\tilde{L}^T}{\mathrm{d}g^N} = -\frac{(1-\alpha_c)}{\tilde{h}} < 0, \tag{87d}$$
$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N} = \frac{(1-\alpha_c)}{\mu_1} < 0, \tag{87e}$$
$$\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^N} = -\frac{\tilde{p}\,(1-\alpha_c)}{\mu_1} > 0. \tag{87f}$$

D Unanticipated Temporary Demand and Supply Disturbances: A Consistent Solution Method for the Two-Sector Model

Following Schubert and Turnovsky [2002], we define a viable steady-state i starting at time $\mathcal{T}_j$ to be one that is consistent with long-run solvency, given the stocks of capital, $K_{\mathcal{T}_j}$ and foreign bonds, $n_{\mathcal{T}_j}$. We rewrite the system of steady-state equations (16) for an arbitrary period j:

$$h_k \left[\tilde{k}^N (\tilde{p}_j)\right] = r^\star, \tag{88a}$$
$$Y^N \left(\tilde{K}_j, \tilde{p}_j\right) - p_c' (\tilde{p}_j)\, \tilde{c}_j - g_j^N = 0, \tag{88b}$$
$$r^\star \tilde{n}_j + Y^T \left(\tilde{K}_j, \tilde{p}_j\right) - (1-\alpha_c)\, p_c (\tilde{p}_j)\, \tilde{c}_j - g_j^T = 0, \tag{88c}$$

together with the intertemporal solvency condition

$$\left(\tilde{n}_j - n_{\mathcal{T}_j}\right) = \frac{N_1}{\mu_1 - r^\star} \left(\tilde{K}_j - K_{\mathcal{T}_j}\right). \tag{88d}$$

Viable steady-state and the two-step method

The new *consistent* procedure consists in two steps. In a **first step**, we solve the system (88a)-(88c) for $\tilde{p}_j$, $\tilde{K}_j$ and $\tilde{n}_j$ as functions of the marginal utility of wealth, $\bar{\lambda}_j$, the government expenditure on the traded and non traded goods. Totally differentiating equations (88a)-(88c) yields in matrix form:

$$\begin{pmatrix} h_{kk} k_p^N & 0 & 0 \\ \left(Y_p^N - c_p^N\right) & Y_K^N & 0 \\ \left(Y_p^T - c_p^T\right) & Y_K^T & r^\star \end{pmatrix} \begin{pmatrix} \mathrm{d}\tilde{p}_j \\ \mathrm{d}\tilde{K}_j \\ \mathrm{d}\tilde{n}_j \end{pmatrix} = \begin{pmatrix} 0 \\ p_c' c_{\bar{\lambda}} \mathrm{d}\bar{\lambda}_j + \mathrm{d}g_j^N \\ (1-\alpha_c)\, p_c c_{\bar{\lambda}} \mathrm{d}\bar{\lambda}_j + \mathrm{d}g_j^T \end{pmatrix} \tag{89}$$

The equilibrium value of the marginal utility of wealth $\bar{\lambda}_j$ and policy parameters, g_j^T, g_j^N, determine the following steady-state values:

$$\tilde{p}_j = \text{constant}, \quad (90a)$$

$$\tilde{K}_j = K\left(\bar{\lambda}_j, g_j^N\right), \quad (90b)$$

$$\tilde{n}_j = v\left(\bar{\lambda}_j, g_j^T, g_j^N\right), \quad (90c)$$

with partial derivatives given by:

$$K_{\bar{\lambda}} \equiv \frac{\partial \tilde{K}_j}{\partial \bar{\lambda}_j} = \frac{h_{kk} k_p^N p_c p_c' r^\star}{G} = -\sigma_c \frac{\tilde{c}_j^N}{\bar{\lambda}_j} \frac{\left(\tilde{k}_j^N - \tilde{k}_j^T\right)}{\tilde{h}_j} \lessgtr 0, \quad (91a)$$

$$\begin{aligned} v_{\bar{\lambda}} \equiv \frac{\partial \tilde{n}_j}{\partial \bar{\lambda}_j} &= \frac{h_{kk} k_p^N p_c \left(-p_c' Y_K^T + (1-\alpha_c) p_c Y_K^N\right)}{G}, \\ &= \frac{p_c \tilde{c}_j}{\bar{\lambda}_j} \frac{\sigma_c}{Y_K^N r^\star} \left[\alpha_c r^\star - \frac{\tilde{h}_j}{\left(\tilde{k}_j^N - \tilde{k}_j^T\right)}\right], \\ &= -\frac{p_c \tilde{c}_j}{\bar{\lambda}_j} \frac{\sigma_c}{r^\star \tilde{p} \tilde{h}_j} \left[\alpha_c \tilde{f}_j + (1-\alpha_c) \tilde{p}_j \tilde{h}_j\right] < 0, \end{aligned} \quad (91b)$$

and

$$K_{g^T} \equiv \frac{\partial \tilde{K}_j}{\partial g_j^T} = 0, \quad (92a)$$

$$v_{g^T} \equiv \frac{\partial \tilde{n}_j}{\partial g_j^T} = \frac{1}{r^\star} > 0, \quad (92b)$$

and

$$K_{g^N} \equiv \frac{\partial \tilde{K}_j}{\partial g_j^N} = \frac{h_{kk} k_p^N u_{cc} r^\star}{G} = \frac{\left(\tilde{k}_j^N - \tilde{k}_j^T\right)}{\tilde{h}_j} \gtrless 0, \quad (93a)$$

$$v_{g^N} \equiv \frac{\partial \tilde{n}_j}{\partial g_j^N} = -\frac{h_{kk} k_p^N u_{cc} Y_K^T}{G} = \frac{\tilde{f}_j}{\tilde{h}_j} \frac{1}{r^\star} > 0, \quad (93b)$$

where $G \equiv h_{kk} k_p^N u_{cc} Y_K^N r^\star$ which simplifies as follows :

$$G \equiv \frac{\tilde{f}\tilde{h}}{\tilde{p}^2 \left(\tilde{k}^N - \tilde{k}^T\right)^2} u_{cc} r^\star < 0. \quad (94)$$

The **second step** consists to determine the equilibrium change of $\bar{\lambda}_j$ by taking the total differential of the intertemporal solvency condition (88d):

$$\begin{aligned} \left[v_{\bar{\lambda}} - \frac{N_1}{\mu_1 - r^\star} K_\lambda\right] \mathrm{d}\bar{\lambda}_j &= \mathrm{d}n_{\mathcal{T}_j} - \frac{N_1}{\mu_1 - r^\star} \mathrm{d}K_{\mathcal{T}_j} - \left[v_{g^N} - \frac{N_1}{\mu_1 - r^\star} K_{g^N}\right] \mathrm{d}g_j^N \\ &- v_{g^T} \mathrm{d}g_j^T, \end{aligned} \quad (95)$$

from which may solve for the equilibrium value of $\bar{\lambda}_j$ as a function of initial stocks at time $\mathcal{T}_j$ and government spending:

$$\bar{\lambda} = \lambda\left(K_{\mathcal{T}_j}, n_{\mathcal{T}_j}, g^T, g^N\right), \quad (96)$$

with

$$\lambda_K \equiv \frac{\partial \bar{\lambda}_j}{\partial K_{\mathcal{T}_j}} = -\frac{\frac{N_1}{\mu_1 - r^\star}}{\left[v_{\bar{\lambda}} - \frac{N_1}{\mu_1 - r^\star} K_{\bar{\lambda}}\right]} < 0, \tag{97a}$$

$$\lambda_n \equiv \frac{\partial \bar{\lambda}_j}{\partial n_{\mathcal{T}_j}} = \frac{1}{\left[v_{\bar{\lambda}} - \frac{N_1}{\mu_1 - r^\star} K_{\bar{\lambda}}\right]} < 0, \tag{97b}$$

$$\lambda_{g^T} \equiv \frac{\partial \bar{\lambda}_j}{\partial g_j^T} = -\frac{v_{g^T}}{\left[v_{\bar{\lambda}} - \frac{N_1}{\mu_1 - r^\star} K_{\bar{\lambda}}\right]} > 0, \tag{97c}$$

$$\lambda_{g^N} \equiv \frac{\partial \bar{\lambda}_j}{\partial g_j^N} = -\frac{\left[v_{g^N} - \frac{N_1}{\mu_1 - r^\star} K_{g^N}\right]}{\left[v_{\bar{\lambda}} - \frac{N_1}{\mu_1 - r^\star} K_{\bar{\lambda}}\right]} > 0. \tag{97d}$$

From (97), we obtain the following properties:

$$\lambda_n \left[v_{\bar{\lambda}} - \frac{N_1}{\mu_1 - r^\star} K_{\bar{\lambda}}\right] = 1, \tag{98a}$$

$$\lambda_n v_{g^T} = -\lambda_{g^T}, \tag{98b}$$

$$\lambda_n \left[v_{g^N} - \frac{N_1}{\mu_1 - r^\star} K_{g^N}\right] = -\lambda_{g^N}. \tag{98c}$$

Formal Solutions for Temporary Demand and Supply Disturbances

We assume that the small open economy is initially in steady-state equilibrium, denoted by the subscript $j = 0$:

$$K_0 = \tilde{K}_0 = K\left(\bar{\lambda}_0, g_0^N\right) = K\left(\lambda\left(K_0, n_0, g_0^T, g_0^N\right), g_0^N\right), \tag{99a}$$

$$n_0 = \tilde{n}_0 = v\left(\bar{\lambda}_0, g_0^T, g_0^N\right) = v\left(\lambda\left(K_0, n_0, g_0^T, g_0^N\right), g_0^T, g_0^N\right), \tag{99b}$$

$$\lambda_0 = \bar{\lambda}_0 = \lambda\left(K_0, n_0, g_0^T, g_0^N\right). \tag{99c}$$

We suppose now that government expenditure changes unexpectedly at time $t = 0$ from the original level g_0^T (resp. g_0^N) to level g_1^T (resp. g_1^N) over the period $0 \leq t < \mathcal{T}$, and reverts back at time $\mathcal{T}$ permanently to its initial level, $g_{\mathcal{T}}^T = g_2^T = g_0^T$ (resp. $g_{\mathcal{T}}^N = g_2^N = g_0^N$).

Period 1 $(0 \leq t < \mathcal{T})$

Whereas the fiscal expansion is implemented, the economy follows unstable transitional paths:

$$K(t) = \tilde{K}_1 + B_1 e^{\mu_1 t} + B_2 e^{\mu_2 t}, \tag{100a}$$

$$p(t) = \tilde{p}_1 + \omega_2^1 B_1 e^{\mu_1 t} + \omega_2^2 B_2 e^{\mu_2 t}, \tag{100b}$$

$$\begin{aligned} n(t) = {} & \tilde{n}_1 + \left[(n_0 - \tilde{n}_1) - \frac{N_1}{\mu_1 - r^\star} B_1 - \frac{N_2}{\mu_2 - r^\star} B_2\right] e^{r^\star t} + \\ & + \frac{N_1}{\mu_1 - r^\star} B_1 e^{\mu_1 t} + \frac{N_2}{\mu_2 - r^\star} B_2 e^{\mu_2 t}, \end{aligned} \tag{100c}$$

with the steady-state values $\tilde{K}_1$ and $\tilde{n}_1$ given by the following functions (determined from (90b)-(90c) with $j = 1$)

$$\tilde{K}_1 = K\left(\bar{\lambda}, g_1^N\right), \tag{101a}$$

$$\tilde{n}_1 = v\left(\bar{\lambda}, g_1^T, g_1^N\right), \tag{101b}$$

where the marginal utility of wealth remains constant over periods 1 and 2 at level $\bar{\lambda}_1 = \bar{\lambda}_2 = \bar{\lambda}$ after its initial jump at time $t = 0$.

Period 2 ($t \geq \mathcal{T}$)

Once government spending reverts back to its initial level, the economy follows stable paths

$$K(t) = \tilde{K}_2 + B_1' e^{\mu_1 t}, \tag{102a}$$

$$p(t) = \tilde{p}_2 + \omega_2^1 B_1' e^{\mu_1 t}, \tag{102b}$$

$$n(t) = \tilde{n}_2 + \frac{N_1}{\mu_1 - r^\star} B_1' e^{\mu_1 t}, \tag{102c}$$

with the steady-state values $\tilde{K}_2$ and $\tilde{n}_2$ given by the following functions (determined from (90b)-(90c) with $j = 2$)

$$\tilde{K}_2 = K\left(\bar{\lambda}, g_2^N\right), \tag{103a}$$

$$\tilde{n}_2 = v\left(\bar{\lambda}, g_2^T, g_2^N\right). \tag{103b}$$

During the transition period 1, the economy accumulates capital and foreign assets. Since this period is unstable, it would lead the nation to violate its intertemporal budget constraint. By contrast, the adjustment process taking place in period 2 is stable and must satisfy the economy's intertemporal budget constraint. At the same time, the zero-root problem requires the equilibrium value of marginal utility of wealth to adjust once-and-for-all when the shock hits the economy. So λ remains constant over the periods 1 and 2. The aim of the *two-step method* is to calculate the deviation of λ such that the country satisfies one single and overall intertemporal budget constraint, given the new relevant initial conditions, $K_{\mathcal{T}}$ and $n_{\mathcal{T}}$, prevailing when the shock ends and accumulated over the unstable period. Therefore, for the country to remain intertemporally solvent, we require:

$$n_{\mathcal{T}} - \tilde{n}_2 = \frac{N_1}{\mu_1 - r^\star}\left(K_{\mathcal{T}} - \tilde{K}_2\right). \tag{104}$$

In order to determine the three constants B_1, B_2, and B_1', and the equilibrium value of marginal utility of wealth, we impose three conditions:

1. Initial conditions $K(0) = K_0$, $n(0) = n_0$ must be met.

2. Economic aggregates K and p remain continuous at time $\mathcal{T}$.

3. The intertemporal solvency constraint (104) must hold implying that the net foreign assets remain continuous at time $\mathcal{T}$.

Set $t = 0$ in solution (100a), equate (100a) and (102a), (100b) and (102b), evaluated at time $t = \mathcal{T}$, one obtains

$$\tilde{K}_1 + B_1 + B_2 = K_0, \tag{105a}$$

$$\tilde{K}_1 + B_1 e^{\mu_1 \mathcal{T}} + B_2 e^{\mu_2 \mathcal{T}} = \tilde{K}_2 + B_1' e^{\mu_1 \mathcal{T}}, \tag{105b}$$

$$\tilde{p}_1 + \omega_2^1 B_1 e^{\mu_1 \mathcal{T}} + \omega_2^2 B_2 e^{\mu_2 \mathcal{T}} = \tilde{p}_2 + \omega_2^1 B_1' e^{\mu_1 \mathcal{T}}, \tag{105c}$$

where we used the continuity condition.

Evaluating $K_{\mathcal{T}}$ and $n_{\mathcal{T}}$ from respectively (100a) and (100c), substituting into (104), and using functions of steady-state values $\tilde{K}_j$ and $\tilde{n}_j$ given by (99) (for $j = 0$), (101) (for $j = 1$), and (103) (for $j = 2$), the intertemporal solvency condition can be rewritten as

$$\begin{aligned}
& v\left(\bar{\lambda}, g_1^T, g_1^N\right) + \left[\left(v\left(\lambda_0, g_0^T, g_0^N\right) - v\left(\bar{\lambda}, g_1^T, g_1^N\right)\right) - \frac{N_1}{\mu_1 - r^\star} B_1 - \frac{N_2}{\mu_2 - r^\star} B_2\right] e^{r^\star \mathcal{T}} \\
& + \frac{N_1}{\mu_1 - r^\star} B_1 e^{\mu_1 \mathcal{T}} + \frac{N_2}{\mu_2 - r^\star} B_2 e^{\mu_2 \mathcal{T}} - v\left(\bar{\lambda}, g_2^T, g_2^N\right) \\
= \; & \frac{N_1}{\mu_1 - r^\star}\left[K\left(\bar{\lambda}, g_1^N\right) + B_1 e^{\mu_1 \mathcal{T}} + B_2 e^{\mu_2 \mathcal{T}} - K\left(\bar{\lambda}, g_2^N\right)\right].
\end{aligned} \tag{106}$$

Then, we approximate the steady-state changes with the differentials by using the dummy notation:

$$\tilde{K}_1 - \tilde{K}_0 \equiv K\left(\bar{\lambda}, g_1^N\right) - K\left(\lambda_0, g_0^N\right) = K_{\bar{\lambda}} \mathrm{d}\bar{\lambda} + K_{g^N} \mathrm{d}g^N, \tag{107a}$$

$$\tilde{K}_2 - \tilde{K}_1 \equiv K\left(\bar{\lambda}, g_2^N\right) - K\left(\bar{\lambda}, g_1^N\right) = -K_{g^N} \mathrm{d}g^N, \tag{107b}$$

$$\tilde{n}_1 - \tilde{n}_0 \equiv v\left(\bar{\lambda}, g_1^T, g_1^N\right) - v\left(\lambda_0, g_0^T, g_0^N\right) = v_{\bar{\lambda}} \mathrm{d}\bar{\lambda} + v_{g^T} \mathrm{d}g^T + v_{g^N} \mathrm{d}g^N, \tag{107c}$$

$$\tilde{n}_2 - \tilde{n}_1 \equiv v\left(\bar{\lambda}, g_2^T, g_2^N\right) - v\left(\bar{\lambda}, g_1^T, g_1^N\right) = -v_{g^T} \mathrm{d}g^T - v_{g^N} \mathrm{d}g^N, \tag{107d}$$

where $\mathrm{d}\bar{\lambda} \equiv \bar{\lambda} - \lambda_0$.

By substituting these expressions in (105) and (106), we obtain finally

$$B_1 + B_2 = -K_{\bar{\lambda}} \mathrm{d}\bar{\lambda} - K_{g^N} \mathrm{d}g^N, \tag{108a}$$

$$B_1 e^{\mu_1 \mathcal{T}} + B_2 e^{\mu_2 \mathcal{T}} - B_1' e^{\mu_1 \mathcal{T}} = -K_{g^N} \mathrm{d}g^N, \tag{108b}$$

$$\omega_2^1 B_1 e^{\mu_1 \mathcal{T}} + \omega_2^2 B_2 e^{\mu_2 \mathcal{T}} - \omega_2^1 B_1' e^{\mu_1 \mathcal{T}} = 0, \tag{108c}$$

and

$$B_1\Upsilon_1 + B_2\Upsilon_2 + v_{\bar{\lambda}}\mathrm{d}\bar{\lambda} = \Phi_1, \tag{109}$$

where we set

$$\Upsilon_1 \equiv \frac{N_1}{\mu_1 - r^\star}, \tag{110a}$$

$$\Upsilon_2 \equiv \frac{N_2}{\mu_2 - r^\star} + \left(\frac{N_1}{\mu_1 - r^\star} - \frac{N_2}{\mu_2 - r^\star}\right)e^{-\mu_1\mathcal{T}}, \tag{110b}$$

$$\Phi_1 \equiv \left[\left(v_{g^j} - \frac{N_1}{\mu_1 - r^\star}K_{g^j}\right)e^{-r^\star\mathcal{T}} - v_{g^j}\right]\mathrm{d}g^j \quad j = T, N, \tag{110c}$$

where $K_{g^T} = 0$.

case $k^N > k^T$

We write out some useful expressions

$$K_{\bar{\lambda}} = -\frac{\tilde{c}^N}{\bar{\lambda}}\frac{\sigma_c}{\mu_2} < 0, \tag{111a}$$

$$K_{g^N} = \frac{1}{\mu_2} > 0, \tag{111b}$$

$$v_{\bar{\lambda}} = -\frac{p_c\tilde{c}}{\bar{\lambda}}\frac{\sigma_c}{\mu_2 r^\star}\left[(1-\alpha_c)\mu_2 - \alpha_c\mu_1\right] < 0, \tag{111c}$$

$$v_{g^N} = -\frac{\tilde{p}\mu_1}{\mu_2 r^\star} > 0, \tag{111d}$$

$$\left(v_{\bar{\lambda}} - \frac{N_1}{\mu_1 - r^\star}K_{\bar{\lambda}}\right) = -\frac{p_c\tilde{c}}{\bar{\lambda}}\frac{\sigma_c}{\mu_2 r^\star}\left[\mu_2 + \alpha_c\frac{r^\star}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right] < 0, \tag{111e}$$

$$\left(v_{g^N} - \frac{N_1}{\mu_1 - r^\star}K_{g^N}\right) = \frac{\tilde{p}}{\mu_2 r^\star}\left[\mu_2 + \frac{r^\star}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right] > 0, \tag{111f}$$

$$\Upsilon_2 = -\tilde{p}\left[1 + \frac{\tilde{c}^N}{\tilde{p}}\frac{\sigma_c}{\mu_2}\omega_2^1 e^{-\mu_1\mathcal{T}}\right], \tag{111g}$$

$$v_{\bar{\lambda}} - \Upsilon_2 K_{\bar{\lambda}} = -\frac{p_c\tilde{c}}{\bar{\lambda}}\frac{\sigma_c}{r^\star\mu_2}\left[\mu_2 + \alpha_c\frac{r^\star}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1 e^{-\mu_1\mathcal{T}}\right] < 0, \tag{111h}$$

$$\Phi_1 K_{\bar{\lambda}} + v_{\bar{\lambda}}K_{g^N}\mathrm{d}g^N = -\frac{p_c\tilde{c}}{\bar{\lambda}}\frac{\sigma_c}{r^\star(\mu_2)^2}\left\{\alpha_c\left[\mu_2 + \frac{r^\star}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]e^{-r^\star\mathcal{T}} + (1-\alpha_c)\mu_2\right\}\mathrm{d}g^N < 0, \tag{111i}$$

and $v_{g^T} = 1/r^\star > 0$. We used the fact that $\tilde{k}^T\mu_2 + \tilde{k}^N\mu_1 = -\frac{w}{\tilde{p}}$ and the following expression:

$$\Phi_1 = -\frac{1}{r^\star}\left(1 - e^{-r^\star\mathcal{T}}\right)\mathrm{d}g^T + \frac{\tilde{p}}{r^\star\mu_2}\left\{\mu_1 + \left[\mu_2 + \frac{r^\star}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]e^{-r^\star\mathcal{T}}\right\}\mathrm{d}g^N. \tag{112}$$

case $k^T > k^N$

We write out some useful expressions

$$\begin{aligned}
K_{\bar{\lambda}} &= -\frac{\tilde{c}^N}{\bar{\lambda}}\frac{\sigma_c}{\mu_1} > 0, && \text{(113a)}\\
K_{g^N} &= \frac{1}{\mu_1} < 0, && \text{(113b)}\\
v_{\bar{\lambda}} &= -\frac{p_c\tilde{c}}{\bar{\lambda}}\frac{\sigma_c}{\mu_1 r^\star}\left[\left(1-\alpha_c\right)\mu_1 - \alpha_c\mu_2\right] < 0, && \text{(113c)}\\
v_{g^N} &= -\frac{\tilde{p}\mu_2}{\mu_1 r^\star} > 0, && \text{(113d)}\\
\left(v_{\bar{\lambda}} - \frac{N_1}{\mu_1 - r^\star}K_{\bar{\lambda}}\right) &= -\frac{p_c\tilde{c}}{\bar{\lambda}}\frac{\sigma_c}{r^\star} < 0 && \text{(113e)}\\
\left(v_{g^N} - \frac{N_1}{\mu_1 - r^\star}K_{g^N}\right) &= \frac{\tilde{p}}{r^\star} > 0, && \text{(113f)}\\
\Upsilon_2 &= -\tilde{p}\left[1 + \frac{\tilde{c}^N}{\tilde{p}}\frac{\sigma_c}{\mu_1}\omega_2^2\left(1 - e^{-\mu_1 \mathcal{T}}\right)\right] < 0, && \text{(113g)}\\
v_{\bar{\lambda}} - \Upsilon_2 K_{\bar{\lambda}} &= -\frac{p_c\tilde{c}}{\bar{\lambda}}\frac{\sigma_c}{r^\star\mu_1}\left[\mu_1 + \alpha_c\frac{r^\star}{\mu_1}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^2\left(1 - e^{-\mu_1\mathcal{T}}\right)\right] \gtrless 0, && \text{(113h)}\\
\Phi_1 K_{\bar{\lambda}} + v_{\bar{\lambda}}K_{g^N}\mathrm{d}g^N &= -\frac{p_c\tilde{c}}{\bar{\lambda}}\frac{\sigma_c}{r^\star\mu_1}\left[\left(1-\alpha_c\right) + \alpha_c e^{-r^\star\mathcal{T}}\right] > 0, && \text{(113i)}
\end{aligned}$$

and $v_{g^T} = 1/r^\star > 0$. We used the fact that $\tilde{k}^T\mu_1 + \tilde{k}^N\mu_2 = -\frac{w}{\tilde{p}}$ and the following expression:

$$\Phi_1 = -\frac{1}{r^\star}\left(1 - e^{-r^\star\mathcal{T}}\right)\mathrm{d}g^T + \frac{\tilde{p}}{r^\star\mu_1}\left(\mu_2 + \mu_1 e^{-r^\star\mathcal{T}}\right)\mathrm{d}g^N. \tag{114}$$

case $k^N > k^T$

The solutions for a rise in the government expenditure on the traded good are given by:

$$\begin{aligned}
\frac{B_1}{\mathrm{d}g^T} &= \frac{\alpha_c\left(1 - e^{-r^\star\mathcal{T}}\right)}{\tilde{p}\mu_2\left[1 + \alpha_c\frac{r^\star}{(\mu_2)^2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]} > 0, && \text{(115a)}\\
\frac{B_2}{\mathrm{d}g^T} &= 0, && \text{(115b)}\\
\frac{B_1'}{\mathrm{d}g^T} &= \frac{B_1}{\mathrm{d}g^T}, && \text{(115c)}\\
\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp} &= \lambda_{g^T}\left(1 - e^{-r^\star\mathcal{T}}\right) > 0, && \text{(115d)}
\end{aligned}$$

where, from (108a), $\frac{B_1}{\mathrm{d}g^T}$ can be written also as follows

$$\frac{B_1}{\mathrm{d}g^T} = -K_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp}. \tag{116}$$

The solutions for a rise in the government expenditure on the non traded good are given by:

$$\begin{aligned}
\frac{B_1}{\mathrm{d}g^N} &= -\frac{\left[\left(1-e^{-\mu_2 \mathcal{T}}\right)-\alpha_c\left(1-e^{-r^\star \mathcal{T}}\right)\right]}{\mu_2\left[1+\alpha_c \frac{r^\star}{(\mu_2)^2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]} \\
&= -\frac{(1-\alpha_c)\left(1-e^{-\mu_2 \mathcal{T}}\right)+\alpha_c\left(e^{-r^\star \mathcal{T}}-e^{-\mu_2 \mathcal{T}}\right)}{\mu_2\left[1+\alpha_c \frac{r^\star}{(\mu_2)^2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]} < 0, && (117a) \\
\frac{B_2}{\mathrm{d}g^N} &= -\frac{e^{-\mu_2 \mathcal{T}}}{\mu_2} < 0, && (117b) \\
\frac{B_1'}{\mathrm{d}g^N} &= \frac{B_1}{\mathrm{d}g^N} < 0, && (117c) \\
\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} &= \left(1-e^{-\mu_2 \mathcal{T}}\right)\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{perm} + \frac{u_{cc}\tilde{p}}{(p_c)^2}\frac{\mu_2\left(e^{-r^\star \mathcal{T}}-e^{-\mu_2 \mathcal{T}}\right)}{\left[\mu_2+\alpha_c\frac{r^\star}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]} \\
&= \lambda_{g^N}\left\{\left(1-e^{-\mu_2 \mathcal{T}}\right)-\frac{\mu_2\left(e^{-r^\star \mathcal{T}}-e^{-\mu_2 \mathcal{T}}\right)}{\left[\mu_2+\frac{r^\star}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]}\right\} \lesseqgtr 0, && (117d)
\end{aligned}$$

where we used expression (86b) to obtain (117d). From (108a), $\frac{B_1}{\mathrm{d}g^T}$ and $\frac{B_2}{\mathrm{d}g^T}$ can also be written as follows:

$$\frac{B_1}{\mathrm{d}g^N}+\frac{B_2}{\mathrm{d}g^N} = -K_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} - K_{g^N} \quad \text{and} \quad \frac{B_2}{\mathrm{d}g^N} = -K_{g^N}e^{-\mu_2 \mathcal{T}}. \tag{118}$$

case $k^T > k^N$

The solutions for a rise in the government expenditure on the traded good are given by:

$$\begin{aligned}
\frac{B_1}{\mathrm{d}g^T} &= \frac{\alpha_c}{\mu_1\tilde{p}}\left(1-e^{-r^\star \mathcal{T}}\right) < 0, && (119a) \\
\frac{B_2}{\mathrm{d}g^T} &= 0, && (119b) \\
\frac{B_1'}{\mathrm{d}g^T} &= \frac{B_1}{\mathrm{d}g^T}, && (119c) \\
\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp} &= \lambda_{g^T}\left(1-e^{-r^\star \mathcal{T}}\right) > 0. && (119d)
\end{aligned}$$

The solutions for a rise in the government expenditure on the non traded good are given by:

$$\begin{aligned}
\frac{B_1}{\mathrm{d}g^N} &= -\frac{1}{\mu_1}\left[(1-\alpha_c)+\alpha_c e^{-r^\star \mathcal{T}}\right], \\
&= -\frac{1}{\mu_1}\left[(1-\alpha_c)\left(1-e^{-r^\star \mathcal{T}}\right)+e^{-r^\star \mathcal{T}}\right] > 0, && (120a) \\
\frac{B_2}{\mathrm{d}g^N} &= 0, && (120b) \\
\frac{B_1'}{\mathrm{d}g^N} &= \frac{B_1}{\mathrm{d}g^N} + K_{g^N}e^{-\mu_1 \mathcal{T}} \\
&= -\frac{1}{\mu_1}\left[\left(1-e^{-\mu_1 \mathcal{T}}\right)-\alpha_c\left(1-e^{-r^\star \mathcal{T}}\right)\right] < 0, && (120c) \\
\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} &= \lambda_{g^N}\left(1-e^{-r^\star \mathcal{T}}\right) > 0. && (120d)
\end{aligned}$$

E Transitional Dynamics

E.1 An Unanticipated Rise in Government Spending on the Traded Good

Steady-State Changes

It is convenient to determine first the long-run changes of the real consumption, the stock of physical capital and the stock of foreign assets following an unanticipated permanent rise in government spending on the traded good by differentiating the functions (40) and (90):

$$\left.\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^T}\right|_{perm} = c_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{perm} < 0, \tag{121a}$$

$$\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T}\right|_{perm} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{perm} \lesseqgtr 0 \quad \text{depending on wether} \quad k^N \gtrless k^T, \tag{121b}$$

$$\left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^T}\right|_{perm} = v_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{perm} + v_{g^T} \gtrless 0 \quad \text{depending on wether} \quad k^N \gtrless k^T, \tag{121c}$$

where $c_{g^T} = 0$ and $K_{g^T} = 0$. Expressions of the steady-state changes are given by the set of equations (84) and (85).

We compare the once-for-all jump of the marginal utility of wealth after an unanticipated permanent increase in public spending on the traded good with respect to its change after a permanent rise:

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp} = \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) = \lambda_{g^T} \left(1 - e^{-r^\star \mathcal{T}}\right) > 0. \tag{122}$$

We now evaluate the long-run changes of key economic variables after an unanticipated temporary public policy by differentiating functions (40) and (90). Since the signs of expressions depend crucially on the sectoral capital intensities, we consider two cases.

Case $k^N > k^T$

When the non traded sector is relatively more capital intensive, the variations of macroeconomic aggregates in the long-run are given by:

$$\left.\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^T}\right|_{temp} = c_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp} = c_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{perm} < 0, \tag{123a}$$

$$\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T}\right|_{temp} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp} = K_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{perm} < 0, \tag{123b}$$

$$\left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^T}\right|_{temp} = v_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp} = v_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{perm} < 0, \tag{123c}$$

where $c_{\bar{\lambda}} < 0$, $K_{\bar{\lambda}} < 0$ (if $k^N > k^T$), and $v_{\bar{\lambda}} < 0$.

The changes of the period 1 steady-state values $\tilde{K}_1$ and $\tilde{n}_1$ compared to their initial (given) values K_0 and n_0 are given by :

$$\left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}g^T}\right|_{temp} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp} < 0, \tag{124a}$$

$$\left.\frac{\mathrm{d}\tilde{n}_1}{\mathrm{d}g^T}\right|_{temp} = v_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp} + v_{g^T} > 0, \tag{124b}$$

where $K_{\bar{\lambda}} < 0$, $v_{\bar{\lambda}} < 0$ and $v_{g^T} > 0$. From (121b)-(121c), (123b)-(123c), and (124a)-(124b), we are able to deduce the following inequalities:

$$\tilde{K}_{perm} < \tilde{K}_1 = \tilde{K}_{temp} < K_0, \tag{125a}$$

$$\tilde{n}_{temp} < n_0 < \tilde{n}_{perm} < \tilde{n}_1. \tag{125b}$$

Case $k^T > k^N$

When the traded sector is relatively more capital intensive, the variations of macroeconomic aggregates in the long-run are given by

$$\left.\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^T}\right|_{temp} = c_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp} = c_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{perm} < 0, \tag{126a}$$

$$\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T}\right|_{temp} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp} = K_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{perm} > 0, \tag{126b}$$

$$\left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^T}\right|_{temp} = v_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp} = v_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{perm} < 0. \tag{126c}$$

It is interesting to compare the magnitudes of the long-run changes in the stock of international assets between a permanent and a temporary fiscal expansion:

$$\left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^T}\right|_{perm} = v_{\bar{\lambda}}\lambda_{g^T} + v_{g^T} \gtreqless v_{\bar{\lambda}}\lambda_{g^T} \left(1 - e^{-r^\star \mathcal{T}}\right) = \left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^T}\right|_{temp}. \tag{127}$$

The key factor that determines the magnitude of the long-run change in the stock of foreign assets is the period of implementation of the government policy. More specifically, there exists a time $\mathcal{T} = \widetilde{\mathcal{T}}$ for which the two changes are equal which is given by

$$\widetilde{\mathcal{T}} = \frac{1}{r^\star} \ln \left[-\frac{v_{\bar{\lambda}}\lambda_{g^T}}{v_{g^T}} \right]. \tag{128}$$

For high durations of the policy, i. e. $\mathcal{T} > \widetilde{\mathcal{T}}$, the deterioration in the net foreign asset position features a greater magnitude after a temporary fiscal expansion compared to a permanent policy. We may summarize our results as follows:

$$\tilde{n}_{temp} < \tilde{n}_{perm} < n_0 \quad \text{if} \quad \mathcal{T} > \widetilde{\mathcal{T}}, \tag{129a}$$

$$\tilde{n}_{perm} < \tilde{n}_{temp} < n_0 \quad \text{if} \quad \mathcal{T} < \widetilde{\mathcal{T}}. \tag{129b}$$

The changes of the period 1 steady-state values $\tilde{K}_1$ and $\tilde{n}_1$ compared to their initial (given) values $\tilde{K}_0$ and $\tilde{n}_0$ are given by :

$$\left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}g^T}\right|_{temp} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp} > 0, \tag{130a}$$

$$\left.\frac{\mathrm{d}\tilde{n}_1}{\mathrm{d}g^T}\right|_{temp} = v_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp} + v_{g^T} \gtreqless 0, \tag{130b}$$

where $K_{\bar{\lambda}} > 0$, $v_{\bar{\lambda}} < 0$ and $v_{g^T} > 0$. The sign of (130b) remains undetermined but we are able to determine the length of the government spending change, denoted by $\bar{\mathcal{T}}$, for which the steady-state state change (130b) turns to be null:

$$\bar{\mathcal{T}} = -\frac{1}{r^\star} \ln \left[\frac{v_{\bar{\lambda}} \lambda_{g^T} + v_{g^T}}{v_{\bar{\lambda}} \lambda_{g^T}} \right]. \tag{131}$$

Existence of $\bar{\mathcal{T}}$ relies upon inequality $v_{\bar{\lambda}} \lambda_{g^T} < v_{\bar{\lambda}} \lambda_{g^T} + v_{g^T} < 0$ which in turn implies that the term in square brackets is positive and less than unity. Consequently, we get the following inequality:

$$\tilde{n}_1 \lesseqgtr n_0 \quad \text{depending on wether} \quad \mathcal{T} \gtreqless \bar{\mathcal{T}}. \tag{132}$$

From (121b)-(121c), (126b)-(126c), (129) and (130a)-(130b), we are able to deduce the following inequalities:

$$K_0 < \tilde{K}_1 = \tilde{K}_{temp} < \tilde{K}_{perm}, \tag{133a}$$

$$\tilde{n}_{perm} < \tilde{n}_{temp} < n_0 \quad \text{if} \quad \mathcal{T} < \tilde{\mathcal{T}}, \tag{133b}$$

$$\tilde{n}_{temp} < \tilde{n}_{perm} < \tilde{n}_0 \quad \text{if} \quad \mathcal{T} > \tilde{\mathcal{T}}, \tag{133c}$$

where we assume that $\tilde{\mathcal{T}} < \bar{\mathcal{T}}$.

Transitional Dynamics after an Unanticipated Permanent Increase in Government Spending on the Traded Good

As we have derived previously, the stable adjustment of the economy is described by a saddle-path in (K, p)-space. The capital stock, the real exchange rate, and the stock of traded bonds evolve according to:

$$K(t) = \tilde{K} + B_1 e^{\mu_1 t}, \tag{134a}$$

$$p(t) = \tilde{p} + \omega_2^1 B_1 e^{\mu_1 t}, \tag{134b}$$

$$n(t) = \tilde{n} + \frac{N_1}{\mu_1 - r^\star} B_1 e^{\mu_1 t}, \tag{134c}$$

where $\omega_2^1 = 0$ if $k^T > k^N$ and with

$$B_1 = K_0 - \tilde{K} = -\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T}\mathrm{d}g^T,$$

where we made use of the constancy of K at time $t = 0$ (i. e. K_0 is predetermined).

case $k^N > k^T$

Using the fact that the steady-state value of the real exchange rate remains affected by an unanticipated permanent rise in g^T, the initial jump of p is given by

$$\left.\frac{\mathrm{d}p(0)}{\mathrm{d}g^T}\right|_{perm} = -\omega_2^1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T}\right|_{perm} < 0. \tag{135}$$

From the short-run static solutions, and by substituting the change in the equilibrium value of the marginal utility of wealth and the initial jump of the real exchange rate, we get the response of the real consumption at time $t = 0$:

$$\begin{aligned}\left.\frac{\mathrm{d}c(0)}{\mathrm{d}g^T}\right|_{perm} &= c_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{perm} + c_p \left.\frac{\mathrm{d}p(0)}{\mathrm{d}g^T}\right|_{perm} = -\frac{\left[1+\alpha_c \frac{1}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c \omega_2^1\right]}{p_c\left[1+\alpha_c \frac{r^\star}{(\mu_2)^2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c \omega_2^1\right]}, \\ &= \left[1+\alpha_c \frac{1}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c \omega_2^1\right]\left.\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^T}\right|_{perm} < 0,\end{aligned} \tag{136}$$

where $0 < \left[1+\alpha_c \frac{1}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c \omega_2^1\right] < 1$. Therefore, we deduce the following inequality

$$\left.\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^T}\right|_{perm} = c_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{perm} < \left.\frac{\mathrm{d}c(0)}{\mathrm{d}g^T}\right|_{perm} < 0. \tag{137}$$

Irrespective of the sectoral capital intensity, a rise in government spending on the traded good induces a once-for-all upward jump of the marginal utility of wealth which reduces the real consumption. When the non-traded sector is more capital intensive, the initial fall of real consumption is moderated by the depreciation of the real exchange rate at time $t = 0$ and falls by less than in the long-run.

Differentiating solutions (134), with respect to time, one obtains:

$$\dot{K}(t) = -\mu_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T}\right|_{perm} e^{\mu_1 t}\mathrm{d}g^T < 0, \tag{138a}$$

$$\dot{p}(t) = -\mu_1 \omega_2^1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T}\right|_{perm} e^{\mu_1 t}\mathrm{d}g^T > 0, \tag{138b}$$

$$\dot{n}(t) = -\mu_1 \frac{N_1}{\mu_1 - r^\star}\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T}\right|_{perm} e^{\mu_1 t}\mathrm{d}g^T > 0, \tag{138c}$$

where $\frac{N_1}{\mu_1 - r^\star} < 0$ and $\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T}\right|_{perm} < 0$.

From equation (38), along the stable adjustment, the real consumption decreases:

$$\dot{c} = -\sigma_c c \alpha_c \frac{\dot{p}}{p} < 0, \tag{139}$$

where $\left(r^\star - \alpha_c \frac{\dot{p}}{p}\right)$ corresponds to the consumption-based real interest rate. After its initial depreciation, the real exchange rate appreciates to revert back to its initial value. This appreciation raises the consumption-based real interest rate above the world interest rate which depresses real consumption.

case $k^T > k^N$

Differentiating solutions (134), with respect to time, one obtains

$$\begin{aligned}
\dot{K}(t) &= -\mu_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T}\right|_{perm} e^{\mu_1 t} \mathrm{d}g^T > 0, && (140a)\\
\dot{p}(t) &= 0, && (140b)\\
\dot{n}(t) &= -\mu_1 \frac{N_1}{\mu_1 - r^\star} \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T}\right|_{perm} e^{\mu_1 t} \mathrm{d}g^T < 0, && (140c)
\end{aligned}$$

where $\frac{N_1}{\mu_1 - r^\star} < 0$ and $\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T}\right|_{perm} > 0$.

Transitional Dynamics after an Unanticipated Temporary Increase in Government Spending on the Traded Good

case $k^N > k^T$

By evaluating formal solution for $p(t)$ and differentiating with respect to g^T, we get the initial jump of p

$$\left.\frac{\mathrm{d}p(0)}{\mathrm{d}g^T}\right|_{temp} = -\omega_2^1 \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T}\right|_{perm} < 0. \tag{141}$$

By adopting a similar procedure, we obtain the initial response of the investment flow following an unanticipated temporary rise in government spending on the traded good :

$$\left.\frac{\mathrm{d}I(0)}{\mathrm{d}g^T}\right|_{temp} = \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}I(0)}{\mathrm{d}g^T}\right|_{perm} < 0. \tag{142}$$

By differentiating the formal solution (100c) over period 1 for $n(t)$ with respect to time, remembering that $B_2/\mathrm{d}g^T = 0$, then evaluating this at $t = 0$, and differentiating with respect to g^T, we obtain the initial response of the current account following a fiscal expansion:

$$\left.\frac{\mathrm{d}ca(0)}{\mathrm{d}g^T}\right|_{temp} = -r^\star \left[\left.\frac{\mathrm{d}\tilde{n}_1}{\mathrm{d}g^T}\right|_{temp} - \frac{N_1}{\mu_1 - r^\star} \left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}g^T}\right|_{temp}\right] + \mu_1 \frac{N_1}{\mu_1 - r^\star} \frac{B_1}{\mathrm{d}g^T}.$$

The expression in brackets can be evaluated by using properties (98), and the fact that $v_{g^T} = -\lambda_{g^T}/\lambda_n$:

$$
\begin{aligned}
-\left[\left.\frac{\mathrm{d}\tilde{n}_1}{\mathrm{d}g^T}\right|_{temp} - \frac{N_1}{\mu_1 - r^\star}\left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}g^T}\right|_{temp}\right] &= -\left[v_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp} + v_{g^T} - \frac{N_1}{\mu_1 - r^\star}K_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp}\right], \\
&= -\left[\frac{\lambda_{g^T}}{\lambda_n}\left(1 - e^{-r^\star \mathcal{T}}\right) - \frac{\lambda_{g^T}}{\lambda_n}\right], \\
&= -v_{g^T}e^{-r^\star \mathcal{T}}.
\end{aligned} \tag{143}
$$

Inserting this expression, and remembering that $\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^T} = \frac{N_1}{\mu_1 - r^\star}\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T}$, we obtain the reaction of the current account at time $t = 0$:

$$
\begin{aligned}
\left.\frac{\mathrm{d}ca(0)}{\mathrm{d}g^T}\right|_{temp} &= -e^{-r^\star \mathcal{T}} - \mu_1\frac{N_1}{\mu_1 - r^\star}K_{\bar{\lambda}}\left(1 - e^{-r^\star \mathcal{T}}\right)\lambda_{g^T}, \\
&= -e^{-r^\star \mathcal{T}} - \mu_1\left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^T}\right|_{perm}\left(1 - e^{-r^\star \mathcal{T}}\right) \lesseqgtr 0.
\end{aligned} \tag{144}
$$

The initial current account response is the result of two conflictory forces: [i] a *smoothing* effect which deteriorates the current account, and [ii] the negative investment flow which improves the external asset position. From (144), there exists a critical value of shock's length, $\hat{\mathcal{T}} > 0$, such that the current account is initially null, i. e. $\dot{n}(0) = 0$. Solving (144) for $\hat{\mathcal{T}}$, one obtains

$$
\hat{\mathcal{T}} = \frac{1}{r^\star}\ln\left[\frac{1 - \mu_1\left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^T}\right|_{perm}}{-\mu_1\left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^T}\right|_{perm}}\right], \tag{145}
$$

where the term in square brackets is higher than one.

The dynamics of the capital stock and the real exchange rate over period 1 are obtained by taking the time derivative of equations (100a) and (100b)

$$
\dot{K}(t) = \mu_1\frac{B_1}{\mathrm{d}g^T}e^{\mu_1 t}\mathrm{d}g^T = -\mu_1\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T}\right|_{perm}\left(1 - e^{-r^\star \mathcal{T}}\right)e^{\mu_1 t}\mathrm{d}g^T < 0, \tag{146a}
$$

$$
\dot{p}(t) = \mu_1\omega_2^1\frac{B_1}{\mathrm{d}g^T}e^{\mu_1 t}\mathrm{d}g^T = -\omega_2^1\mu_1\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T}\right|_{perm}\left(1 - e^{-r^\star \mathcal{T}}\right)e^{\mu_1 t}\mathrm{d}g^T > 0, \tag{146b}
$$

where we used the fact that $B_1/\mathrm{d}g^T = -\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T}\right|_{perm}\left(1 - e^{-r^\star \mathcal{T}}\right)$.

While the real exchange rate and the stock of physical capital evolve in the same direction than after an unanticipated permanent rise in government spending on the traded good, the differentiation with respect to time of solution (100c) indicates that the current account may change of sign over period 1:

$$
ca(t) = \dot{n}(t) = -e^{-r^\star(\mathcal{T}-t)}\mathrm{d}g^T - \mu_1\left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^T}\right|_{perm}\left(1 - e^{-r^\star \mathcal{T}}\right)e^{\mu_1 t}\mathrm{d}g^T \lesseqgtr 0. \tag{147}
$$

We have now to determine the conditions under which the current account dynamics displays a non monotonic behavior. Equation (147) reveals that the stock of international assets reaches a turning point during its transitional adjustment at time $\hat{\mathcal{T}}$ given by

$$\hat{\mathcal{T}} = \frac{1}{\mu_2} \ln \left[-\mu_1 \frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^T}|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) e^{r^\star \mathcal{T}} \right]. \tag{148}$$

The necessary condition for $\hat{\mathcal{T}} > 0$, corresponds to:

$$0 < e^{-r^\star \mathcal{T}} < -\mu_1 \left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^T}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) \quad \Leftrightarrow \quad \left.\frac{\mathrm{d}ca(0)}{\mathrm{d}g^T}\right|_{temp} > 0. \tag{149}$$

If the fiscal expansion lasts a short period, i. e. $\mathcal{T} < \hat{\mathcal{T}}$, the current account initially deteriorates and the stock of foreign assets decreases monotonically until time $\mathcal{T}$. If the fiscal expansion lasts a time period longer than $\hat{\mathcal{T}}$, the current account initially improves before reaching a turning point at time $\hat{\mathcal{T}}$. Subsequently, the current account deteriorates until time $\mathcal{T}$.

Once the government policy has been removed at time $\mathcal{T}$, the real exchange rate keeps on depreciating and the capital stock converges towards its new lower steady-state value:

$$\dot{K}(t) = \mu_1 \frac{B_1'}{\mathrm{d}g^T} e^{\mu_1 t} \mathrm{d}g^T < 0, \tag{150a}$$

$$\dot{p}(t) = \mu_1 \omega_2^1 \frac{B_1'}{\mathrm{d}g^T} e^{\mu_1 t} \mathrm{d}g^T > 0, \tag{150b}$$

where $B_1'/\mathrm{d}g^T = B_1/\mathrm{d}g^T > 0$. Over period 2, the current account improves unambiguously as it can be seen from the time derivative of solution (102c):

$$\dot{n}(t) = \mu_1 \frac{N_1}{\mu_1 - r^\star} \frac{B_1'}{\mathrm{d}g^T} e^{\mu_1 t} \mathrm{d}g^T > 0. \tag{151}$$

case $k^T > k^N$

When the traded sector is relatively more capital intensive, the real exchange rate dynamics are flat like after a permanent fiscal expansion since the constant $B_2/\mathrm{d}g^T$ is null, i.e. $\dot{p}(t) = 0$. The investment flow is positive over period 1

$$I(t) = \dot{K}(t) = \mu_1 \frac{B_1}{\mathrm{d}g^T} e^{\mu_1 t} \mathrm{d}g^T = -\mu_1 K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp} e^{\mu_1 t} \mathrm{d}g^T > 0. \tag{152}$$

By differentiating the formal solution (100c) over period 1 with respect to time and remembering that $B_2/\mathrm{d}g^T = 0$, one obtains the transitional path for $n(t)$:

$$ca(t) = -r^\star \left[\left.\frac{\mathrm{d}\tilde{n}_1}{\mathrm{d}g^T}\right|_{temp} - \frac{N_1}{\mu_1 - r^\star} \left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}g^T}\right|_{temp} \right] e^{r^\star t} \mathrm{d}g^T + \mu_1 \frac{N_1}{\mu_1 - r^\star} \frac{B_1}{\mathrm{d}g^T} e^{\mu_1 t} \mathrm{d}g^T. \tag{153}$$

By evaluating this expressions at $t = 0$, and differentiating with respect to g^T, we obtain the initial response of the current account following a fiscal expansion:

$$\left.\frac{\mathrm{d}ca(0)}{\mathrm{d}g^T}\right|_{temp} = -r^\star \left[\left.\frac{\mathrm{d}\tilde{n}_1}{\mathrm{d}g^T}\right|_{temp} - \frac{N_1}{\mu_1 - r^\star}\left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}g^T}\right|_{temp}\right] + \mu_1 \frac{N_1}{\mu_1 - r^\star}\frac{B_1}{\mathrm{d}g^T}.$$

The expression in brackets can be evaluated by using properties (98), and the fact that $v_{g^T} = -\lambda_{g^T}/\lambda_n$:

$$\begin{aligned}
-\left[\left.\frac{\mathrm{d}\tilde{n}_1}{\mathrm{d}g^T}\right|_{temp} - \frac{N_1}{\mu_1 - r^\star}\left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}g^T}\right|_{temp}\right] &= -\left[v_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp} + v_{g^T} - \frac{N_1}{\mu_1 - r^\star}K_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp}\right], \\
&= -\left[\frac{\lambda_{g^T}}{\lambda_n}\left(1 - e^{-r^\star \mathcal{T}}\right) - \frac{\lambda_{g^T}}{\lambda_n}\right], \\
&= -v_{g^T} e^{-r^\star \mathcal{T}}. \qquad (154)
\end{aligned}$$

Inserting expression (154) and remembering that $\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^T} = \frac{N_1}{\mu_1 - r^\star}\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T}$, we obtain the reaction of the current account at time $t = 0$:

$$\begin{aligned}
\left.\frac{\mathrm{d}ca(0)}{\mathrm{d}g^T}\right|_{temp} &= -e^{-r^\star \mathcal{T}} - \mu_1 \frac{N_1}{\mu_1 - r^\star} K_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right)\lambda_{g^T}, \\
&= -\left[e^{-r^\star \mathcal{T}} + \mu_1 \left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^T}\right|_{perm}\left(1 - e^{-r^\star \mathcal{T}}\right)\right] < 0. \qquad (155)
\end{aligned}$$

If $k^T > k^N$, both the *smoothing* effect and the positive investment flow lead to a decumulation of foreign assets. Consequently, the current account deteriorates initially and the stock of internationally traded bonds keeps on decreasing over period 1:

$$ca(t) = \dot{n}(t) = -e^{-r^\star(\mathcal{T}-t)}\mathrm{d}g^T - \mu_1 \left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^T}\right|_{perm}\left(1 - e^{-r^\star \mathcal{T}}\right)e^{\mu_1 t}\mathrm{d}g^T < 0. \qquad (156)$$

Over period 2, the stocks of physical capital keeps on decreasing and the current account deteriorates monotonically:

$$\begin{aligned}
I(t) &= \mu_1 \frac{B_1'}{\mathrm{d}g^T} e^{\mu_1 t}\mathrm{d}g^T > 0, \qquad (157a) \\
ca(t) &= \mu_1 \frac{N_1}{\mu_1 - r^\star}\frac{B_1'}{\mathrm{d}g^T} e^{\mu_1 t}\mathrm{d}g^T < 0, \qquad (157b)
\end{aligned}$$

where $B_1'/\mathrm{d}g^T = B_1/\mathrm{d}g^T < 0$.

E.2 An Unanticipated Rise in Government Spending on the Non Traded Good

Steady-State Changes

We derive the ultimate steady-state changes of the economic key variables after an unanticipated permanent rise in government spending on the non traded good by differentiating the functions (90) w.r.t g^N:

$$\left.\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^N}\right|_{perm} = c_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{perm} < 0, \tag{158a}$$

$$\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N}\right|_{perm} = K_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{perm} + K_{g^N} \gtrless 0 \quad \text{depending on wether} \quad k^N \gtrless k^T, \tag{158b}$$

$$\left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^N}\right|_{perm} = v_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{perm} + v_{g^N} \lessgtr 0 \quad \text{depending on wether} \quad k^N \gtrless k^T, \tag{158c}$$

where analytical expressions are given by the set of equations (86) and (87).

We turn now to the long-run changes of macroeconomic aggregates after an unanticipated temporary fiscal expansion by considering two cases.

case $k^N > k^T$

The equilibrium change of $\bar{\lambda}$ is:

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} = \lambda_{g^N}\left\{\left(1 - e^{-\mu_2 \mathcal{T}}\right) - \frac{\mu_2\left(e^{-r^\star \mathcal{T}} - e^{-\mu_2 \mathcal{T}}\right)}{\left[\mu_2 + \frac{r^\star}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]}\right\} < 0. \tag{159}$$

The sign of the change in the equilibrium value of the marginal utility of wealth can be determined by noting that expression (159) tends towards zero whenever the parameter $\mathcal{T}$ tends towards *zero* and tends towards λ_{g^N} when the parameter tends towards ∞. In addition, the term in square brackets is an increasing and monotonic function of parameter $\mathcal{T}$. Therefore, the change in $\bar{\lambda}$ after an unanticipated temporary rise in government spending lies in the range $\left[0, \lambda_{g^N}\right]$. Consequently, we can deduce that expression (159) has a positive sign.

Using the functions (90), we deduce the long-run changes for the real consumption, the stock of physical capital, and the stock of traded bonds:

$$\left.\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^N}\right|_{temp} = c_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} < 0, \tag{160a}$$

$$\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N}\right|_{temp} = K_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} < 0, \tag{160b}$$

$$\left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^N}\right|_{temp} = v_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} < 0, \tag{160c}$$

where $c_{\bar{\lambda}} < 0$, $K_{\bar{\lambda}} < 0$, and $v_{\bar{\lambda}} < 0$.

The change of the period 1 steady-state value $\tilde{K}_1$ compared to its initial (given) value $\tilde{K}_0$ is given by :

$$\left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}g^N}\right|_{temp} = K_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} + K_{g^N},$$
$$= \frac{(1-\alpha_c) + \alpha_c\left[1 + \frac{r^\star}{(\mu_2)^2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1 e^{\mu_1 \mathcal{T}}\right]e^{-r^\star \mathcal{T}}}{\mu_2\left[1+\alpha_c\frac{r^\star}{(\mu_2)^2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]} > 0, \tag{161}$$

where we have substituted the expressions of $K_{\bar{\lambda}} < 0$ given by (111a), $\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\Big|_{temp} > 0$ given by (159) and $K_{g^N} > 0$ given by (111b).

The change of the period 1 steady-state value $\tilde{n}_1$ compared to its initial (given) value $\tilde{n}_0$ is given by :

$$\left.\frac{\mathrm{d}\tilde{n}_1}{\mathrm{d}g^N}\right|_{temp} = v_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} + v_{g^N}, \tag{162}$$
$$= -\frac{\tilde{p}}{r^*\mu_2}\frac{1}{\left[1+\alpha_c\frac{r^*}{(\mu_2)^2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]}\left\{\left((1-\alpha_c)\mu_2 - \alpha_c\mu_1\right)\left[1+\frac{r^*}{(\mu_2)^2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]\left(1-e^{-r^*\mathcal{T}}\right)\right.$$
$$\left. + \left[1+\alpha_c\frac{r^*}{(\mu_2)^2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]\mu_1\right\} \gtrless 0,$$

where we have substituted the expressions of $v_{\bar{\lambda}} < 0$ given by (111c), $\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} > 0$ given by (159) and $v_{g^N} > 0$ given by (111d). The sign of expression (162) remains undetermined because it is the sum of two terms of opposite signs. The first term on the right-hand side of (162) is negative and is an increasing function of parameter $\mathcal{T}$ and may be dominated by the second term v_{g^N} which is positive. We may infer that the shorter-lasting the rise in government expenditure, the more likely a higher steady-state value $\tilde{n}_1$ compared to its initial (given) value $\tilde{n}_0$.

It is interesting to compare the magnitudes of the long-run changes in the stock of international assets between a permanent and a temporary fiscal expansion:

$$\left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^N}\right|_{perm} = v_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{perm} + v_{g^N} \gtrless v_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} = \left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^N}\right|_{temp}, \tag{163}$$

where $v_{g^N} > 0$, $v_{\bar{\lambda}} < 0$ and $\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{perm} = \lambda_{g^N} > 0$. The key factor that determines the magnitude of the long-run change in the stock of foreign assets is the period of implementation of the government policy. More specifically, simulations indicate that there exists a time $\mathcal{T} = \acute{\mathcal{T}}$ for which the two changes are equal. For high durations of the policy, i. e. $\mathcal{T} > \acute{\mathcal{T}}$, the deterioration of the net foreign asset position features a greater magnitude after a temporary fiscal expansion compared to a permanent policy. This result is reversed when the public policy is implemented over a short period, say $\mathcal{T} < \acute{\mathcal{T}}$.

From steady-state changes following unanticipated permanent and temporary rise in government expenditure falling on the non traded good, we can deduce the following inequalities whatever the duration of the public policy:

$$\tilde{K}_{temp} < K_0 < \tilde{K}_{perm} < \tilde{K}_1, \tag{164a}$$

$$\tilde{n}_{temp} < \tilde{n}_{perm} < n_0, \quad \text{if} \quad \mathcal{T} > \acute{\mathcal{T}}, \tag{164b}$$

$$\tilde{n}_{perm} < \tilde{n}_{temp} < n_0, \quad \text{if} \quad \mathcal{T} < \acute{\mathcal{T}}. \tag{164c}$$

case $k^T > k^N$

The equilibrium change of $\bar{\lambda}$ is:

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} = \lambda_{g^N}\left(1 - e^{-r^\star \mathcal{T}}\right) > 0. \tag{165}$$

From (165), we see that that change of λ for a temporary change in g^N is smaller than after a permanent increase in g^N but in the same direction. Hence we deduce the following inequality:

$$0 < \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} < \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{perm}. \tag{166}$$

From the functions (90), we deduce the long-run changes for the real consumption, the stock of physical capital, and the stock of traded bonds:

$$\left.\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^N}\right|_{temp} = c_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} < 0, \tag{167a}$$

$$\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N}\right|_{temp} = K_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} > 0, \tag{167b}$$

$$\left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^N}\right|_{temp} = v_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} < 0, \tag{167c}$$

where $c_{\bar{\lambda}} < 0$, $K_{\bar{\lambda}} > 0$, and $v_{\bar{\lambda}} < 0$.

The changes of the period 1 steady-state values $\tilde{K}_1$ and $\tilde{n}_1$ compared to their initial (given) values K_0 and n_0 are given by :

$$\begin{aligned}
\left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}g^N}\right|_{temp} &= K_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} + K_{g^N}, \\
&= \frac{(1-\alpha_c) + \alpha_c e^{-r^\star \mathcal{T}}}{\mu_1} < 0, \qquad (168a)\\
\left.\frac{\mathrm{d}\tilde{n}_1}{\mathrm{d}g^N}\right|_{temp} &= v_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} + v_{g^N}, \\
&= -\frac{\tilde{p}}{r^\star \mu_1}\left\{(1-\alpha_c)\, r^\star - \left[(1-\alpha_c)\,\mu_1 - \alpha_c \mu_2\right] e^{-r^\star \mathcal{T}}\right\} > 0. \qquad (168b)
\end{aligned}$$

where we have evaluated the signs of (168a)-(168b) by making use of (113a)-(113d) and (87b).

From (166), because the change in the equilibrium value of $\bar{\lambda}$ following a temporary change in g^N is moderated compared to a permanent increase in government expenditure g^N, by making use of (167b)-(167c), (158b)-(158c), and (168a)-(168b), we are able to infer the following inequalities:

$$\tilde{K}_1 < \tilde{K}_{perm} < K_0 < \tilde{K}_{temp}, \tag{169a}$$

$$\tilde{n}_{temp} < n_0 < \tilde{n}_{perm} < \tilde{n}_1. \tag{169b}$$

Transitional Dynamics after an Unanticipated Permanent Increase in Government Spending on the Non Traded Good

Case $k^N > k^T$

The initial jump of p is obtained by setting $t = 0$ in (62b) and then by differentiating with respect to g^N:

$$\left.\frac{\mathrm{d}p(0)}{\mathrm{d}g^N}\right|_{perm} = -\omega_2^1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N}\right|_{perm} > 0. \tag{170}$$

From the short-run static solutions, and by substituting the change in the equilibrium value of the marginal utility of wealth and the initial jump of the real exchange rate, we get the initial jump of real consumption:

$$\begin{aligned} \left.\frac{\mathrm{d}c(0)}{\mathrm{d}g^N}\right|_{perm} &= c_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{perm} + c_p \left.\frac{\mathrm{d}p(0)}{\mathrm{d}g^N}\right|_{perm} = -\frac{\tilde{p}\left[\mu_2 + \frac{r^\star}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right] - \tilde{c}^N\sigma_c\omega_2^1}{p_c\left[\mu_2 + \alpha_c\frac{r^\star}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]} \\ &= \left.\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^N}\right|_{perm} + \frac{(1-\alpha_c)}{p_c}\frac{\tilde{c}^N\sigma_c\omega_2^1}{\left[\mu_2 + \alpha_c\frac{r^\star}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]} < 0. \end{aligned} \tag{171}$$

From (171), we deduce the following inequality

$$\left.\frac{\mathrm{d}c(0)}{\mathrm{d}g^N}\right|_{perm} < \left.\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^N}\right|_{perm} = c_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{perm} < 0. \tag{172}$$

The rise in the marginal utility of wealth and the initial appreciation in the real exchange rate lowers $c(0)$ below its steady-state value. From equation (38), along the stable adjustment, the real consumption rises:

$$\dot{c}(t) = -c\sigma_c\alpha_c\frac{\dot{p}(t)}{p(t)} > 0, \tag{173}$$

where the real exchange rate depreciates along the stable adjustment when the non traded sector is relatively more capital intensive. Otherwise, the real exchange rate's and thus the real consumption's temporal paths are flat.

The dynamics of the key economic variables after a permanent rise in government spending falling on the non traded good are as follows:

$$\dot{K}(t) = -\mu_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N}\right|_{perm} e^{\mu_1 t}\mathrm{d}g^N > 0, \tag{174a}$$

$$\dot{p}(t) = -\mu_1 \omega_2^1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N}\right|_{perm} e^{\mu_1 t}\mathrm{d}g^N < 0, \tag{174b}$$

$$\dot{n}(t) = -\mu_1 \frac{N_1}{\mu_1 - r^\star} \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N}\right|_{perm} e^{\mu_1 t}\mathrm{d}g^N < 0. \tag{174c}$$

Note that the long-run changes for $\tilde{K}$ and $\tilde{n}$ are reversed compared to a permanent rise in government spending falling on the traded good and thus dynamics.

Case $k^T > k^N$

If $k^T > k^N$, the initial change in the real consumption is solely affected by the change in the equilibrium value of the marginal utility of wealth and jumps immediately to its new lower steady-state level:

$$\left.\frac{\mathrm{d}c(0)}{\mathrm{d}g^N}\right|_{perm} = c_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{perm} = \left.\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^N}\right|_{perm} < 0. \tag{175}$$

Over time, investment decreases and the stock of international assets rises:

$$I(t) = -\mu_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N}\right|_{perm} e^{\mu_1 t}\mathrm{d}g^N < 0, \tag{176a}$$

$$ca(t) = -\mu_1 \frac{N_1}{\mu_1 - r^\star} \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N}\right|_{perm} e^{\mu_1 t}\mathrm{d}g^N > 0. \tag{176b}$$

Since it will be useful later, we calculate the slope of the trajectory after a permanent fiscal expansion in the (K, n)-space by differentiating the solutions for $n(t)$ and for $K(t)$ w.r.t time and dividing the resulting expressions:

$$\frac{\mathrm{d}n(t)}{\mathrm{d}K(t)} = \frac{\mu_1 \frac{N_1}{\mu_1 - r^\star} \frac{B_1}{\mathrm{d}g^N} e^{\mu_1 t}}{\mu_1 \frac{B_1}{\mathrm{d}g^N} e^{\mu_1 t}} = -\tilde{p} < 0. \tag{177}$$

Transitional Dynamics after an Unanticipated Temporary Increase in Government Spending on the Non Traded Good

case $k^N > k^T$

First, we evaluate the constants $B_1/\mathrm{d}g^N$ and $B_2/\mathrm{d}g^N$:

$$\frac{B_1}{\mathrm{d}g^N} = -\frac{B_2}{\mathrm{d}g^N} - K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} - K_{g^N},$$

$$= -\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T}\right|_{perm} \left[\left(1 - e^{-\mu_2 \mathcal{T}}\right) + \left(\frac{\alpha_c}{1-\alpha_c}\right)\left(e^{-r^\star \mathcal{T}} - e^{-\mu_2 \mathcal{T}}\right)\right] < 0. \tag{178a}$$

$$\frac{B_2}{\mathrm{d}g^N} = -K_{g^N} e^{-\mu_2 \mathcal{T}} = -\frac{e^{-\mu_2 \mathcal{T}}}{\mu_2} < 0. \tag{178b}$$

By evaluating the formal solution for $p(t)$ at time $t = 0$, differentiating with respect to g^N, and remembering that $\mathrm{d}\tilde{p}_1/\mathrm{d}g^N = 0$, we get the initial jump of p:

$$\begin{aligned}
\left.\frac{\mathrm{d}p(0)}{\mathrm{d}g^N}\right|_{temp} &= \omega_2^1 \frac{B_1}{\mathrm{d}g^N} = -\omega_2^1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N}\right|_{perm} \left(1 - e^{-\mu_2 \mathcal{T}}\right) - \omega_2^1 \frac{\alpha_c \left(e^{-r^\star \mathcal{T}} - e^{-\mu_2 \mathcal{T}}\right)}{\left[\mu_2 + \frac{r^\star}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c \omega_2^1\right]} \\
&= -\omega_2^1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N}\right|_{perm} \left[\left(1 - e^{-\mu_2 \mathcal{T}}\right) + \left(\frac{\alpha_c}{1-\alpha_c}\right)\left(e^{-r^\star \mathcal{T}} - e^{-\mu_2 \mathcal{T}}\right)\right] > 0, \quad (179)
\end{aligned}$$

where we have inserted the steady-state change of the capital stock after a permanent fiscal expansion falling on the non-traded good given by (86e). From (179), we can see that the magnitude of the initial appreciation in the real exchange after a temporary fiscal expansion may be magnified if the policy is implemented during a long period, i. e. for $T > \frac{1}{\mu_1} \ln\left[\alpha_c\right]$.

By making use of the short-run static solution (40) for c, we get the response of the real consumption at time $t = 0$:

$$\left.\frac{\mathrm{d}c(0)}{\mathrm{d}g^N}\right|_{temp} = c_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} + c_p \left.\frac{\mathrm{d}p(0)}{\mathrm{d}g^N}\right|_{temp} < 0. \quad (180)$$

It is now convenient to evaluate the magnitude of the downward jump of real consumption after a temporary rise in g^N compared to its size after a permanent public policy by computing the following expression:

$$\left.\frac{\mathrm{d}c(0)}{\mathrm{d}g^N}\right|_{temp} - \left.\frac{\mathrm{d}c(0)}{\mathrm{d}g^N}\right|_{perm} = c_{\bar{\lambda}} \left[\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} - \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{perm}\right] + c_p \left[\left.\frac{\mathrm{d}p(0)}{\mathrm{d}g^N}\right|_{temp} - \left.\frac{\mathrm{d}p(0)}{\mathrm{d}g^N}\right|_{perm}\right] \gtrless 0. \quad (181)$$

The initial response of the investment flow following an unanticipated temporary rise in government spending on the non-traded good is given by:

$$\begin{aligned}
\left.\frac{\mathrm{d}I(0)}{\mathrm{d}g^N}\right|_{temp} &= \mu_1 \frac{B_1}{\mathrm{d}g^N} + \mu_2 \frac{B_2}{\mathrm{d}g^N} \\
&= -\mu_1 \left\{\frac{\left(1-\alpha_c\right)\left(1 - e^{-\mu_2 \mathcal{T}}\right) + \alpha_c \left(e^{-r^\star \mathcal{T}} - e^{-\mu_2 \mathcal{T}}\right)}{\left[\mu_2 + \alpha_c \frac{r^\star}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c \omega_2^1\right]}\right\} - e^{-\mu_2 \mathcal{T}}, \\
&= -\mu_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N}\right|_{perm} \left[\left(1 - e^{-\mu_2 \mathcal{T}}\right) + \left(\frac{\alpha_c}{1-\alpha_c}\right)\left(e^{-r^\star \mathcal{T}} - e^{-\mu_2 \mathcal{T}}\right)\right] - e^{-\mu_2 \mathcal{T}} \gtrless 0.
\end{aligned} \quad (182)$$

The sign of expression (182) is not clear-cut. As investment plays the role of clearing the non traded goods market, its sign depends on the jumps of the real exchange rate and of the marginal utility of wealth. More specifically, the real exchange rate appreciates which raises in turn the production of non traded and decreases their consumption. This effect exerts a positive influence on investment. But in the same time, if public spending lasts a short period, the marginal utility of wealth may jump downward which affects positively c^N and may more than outweigh the first effect which leads to a decumulation of capital.

By differentiating the formal solution (100c) over period 1 for $n(t)$ with respect to time, then evaluating the resulting expressions at $t = 0$, and differentiating with respect to g^N, we obtain the initial response of the current account following an unanticipated temporary fiscal expansion:

$$\begin{aligned} \left.\frac{\mathrm{d}ca(0)}{\mathrm{d}g^N}\right|_{temp} &= r^{\star}\left\{-\left.\frac{\mathrm{d}\tilde{n}_1}{\mathrm{d}g^N}\right|_{temp} - \frac{N_1}{\mu_1 - r^{\star}}\frac{B_1}{\mathrm{d}g^N} - \frac{N_2}{\mu_2 - r^{\star}}\frac{B_2}{\mathrm{d}g^N}\right\} \\ &\quad + \mu_1\frac{N_1}{\mu_1 - r^{\star}}\frac{B_1}{\mathrm{d}g^N} + \mu_2\frac{N_2}{\mu_2 - r^{\star}}\frac{B_2}{\mathrm{d}g^N}. \end{aligned} \tag{183}$$

In order to simplify the solution (183), we rewrite the term in square brackets as follows

$$\begin{aligned} & -\left.\frac{\mathrm{d}\tilde{n}_1}{\mathrm{d}g^N}\right|_{temp} - \left[\frac{N_1}{\mu_1 - r^{\star}}\frac{B_1}{\mathrm{d}g^N} + \frac{N_2}{\mu_2 - r^{\star}}\frac{B_2}{\mathrm{d}g^N}\right] \\ = & -\left[v_{\bar{\lambda}} - \frac{N_1}{\mu_1 - r^{\star}}K_{\bar{\lambda}}\right]\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} - \left[v_{g^N} - \frac{N_1}{\mu_1 - r^{\star}}K_{g^N}\right] + \left[\frac{N_1}{\mu_1 - r^{\star}} - \frac{N_2}{\mu_2 - r^{\star}}\right]\frac{B_2}{\mathrm{d}g^N}, \\ = & -\frac{\lambda_{g^N}}{\lambda_n}\left\{\left(1 - e^{-\mu_2 \mathcal{T}}\right) - \frac{\mu_2\left(e^{-r^{\star}\mathcal{T}} - e^{-\mu_2\mathcal{T}}\right)}{\left[\mu_2 + \frac{r^{\star}}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]}\right\} + \frac{\lambda_{g^N}}{\lambda_n} + \frac{1}{\mu_2}\tilde{c}^N\sigma_c\omega_2^1 K_{g^N}e^{-\mu_2\mathcal{T}}, \\ = & \frac{\lambda_{g^N}}{\lambda_n}e^{\mu_2\mathcal{T}} - \frac{\tilde{p}}{r^{\star}}e^{-r^{\star}\mathcal{T}} + \frac{\tilde{p}}{\mu_2 r^{\star}}\left[\mu_2 + \frac{r^{\star}}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]e^{-\mu_2\mathcal{T}}, \\ = & -\frac{\tilde{p}}{r^{\star}}e^{-r^{\star}\mathcal{T}} < 0, \end{aligned} \tag{184}$$

where we have substituted the expression of the change in the equilibrium value of the marginal utility of wealth given by (117d), we made use of properties (98), expression (111f) and inserted these useful expressions:

$$\begin{aligned} \frac{B_1}{\mathrm{d}g^N} &= -\frac{B_2}{\mathrm{d}g^N} - K_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} - K_{g^N} < 0, \\ \frac{N_1}{\mu_1 - r^{\star}} - \frac{N_2}{\mu_2 - r^{\star}} &= -\frac{1}{\mu_2}\tilde{c}^N\sigma_c\omega_2^1 > 0, \\ \frac{B_2}{\mathrm{d}g^N} &= -K_{g^N}e^{-\mu_2\mathcal{T}} < 0, \\ \frac{\left(v_{g^N} - \frac{N_1}{\mu_1 - r^{\star}}K_{g^N}\right)}{\left[\mu_2 + \frac{r^{\star}}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c\omega_2^1\right]} &= \frac{\tilde{p}}{\mu_2 r^{\star}} > 0. \end{aligned}$$

By inserting (184) into (183), the expression of the initial response of the current account reduces to:

$$
\begin{aligned}
\left.\frac{\mathrm{d}ca(0)}{\mathrm{d}g^N}\right|_{temp} &= -\tilde{p}e^{-r^\star \mathcal{T}} + \mu_1 \tilde{p}\left(1+\frac{1}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c \omega_2^1\right)\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N}\right|_{perm}\left[\left(1-e^{-\mu_2 \mathcal{T}}\right)+\left(\frac{\alpha_c}{1-\alpha_c}\right)\left(e^{-r^\star \mathcal{T}}-e^{-\mu_2 \mathcal{T}}\right)\right] \\
&\quad + \tilde{p}e^{-\mu_2 \mathcal{T}}, \\
&= -\tilde{p}\left(1+\frac{1}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c \omega_2^1\right)\left.\frac{\mathrm{d}I(0)}{\mathrm{d}g^N}\right|_{perm}\left[\left(1-e^{-\mu_2 \mathcal{T}}\right)+\left(\frac{\alpha_c}{1-\alpha_c}\right)\left(e^{-r^\star \mathcal{T}}-e^{-\mu_2 \mathcal{T}}\right)\right] \\
&\quad -\tilde{p}\left(e^{-r^\star \mathcal{T}}-e^{-\mu_2 \mathcal{T}}\right) < 0,
\end{aligned} \tag{185}
$$

where we simplified several expressions as follows:

$$
\begin{aligned}
K_{\bar{\lambda}}\frac{u_{cc}\tilde{p}}{p_c^2}\mu_2 &= \frac{\tilde{p}\tilde{c}^N}{p_c \tilde{c}} = \alpha_c > 0, \\
\mu_2 \frac{N_2}{\mu_2 - r^\star} - \mu_1 \frac{N_1}{\mu_1 - r^\star} &= -\tilde{p}\mu_2 + \tilde{p}\mu_1\left(1+\frac{1}{\mu_2}\frac{\tilde{c}^N}{\tilde{p}}\sigma_c \omega_2^1\right) < 0.
\end{aligned}
$$

Now, we investigate the investment and the real exchange rate dynamics over the unstable period $(0, \mathcal{T})$, say period 1:

$$
\dot{K}(t) = \mu_1 \frac{B_1}{\mathrm{d}g^N}e^{\mu_1 t}\mathrm{d}g^N + \mu_2 \frac{B_2}{\mathrm{d}g^N}e^{\mu_2 t}\mathrm{d}g^N \gtreqless 0, \tag{186a}
$$

$$
\dot{p}(t) = \mu_1 \omega_2^1 \frac{B_1}{\mathrm{d}g^N}e^{\mu_1 t}\mathrm{d}g^N < 0, \tag{186b}
$$

where $B_1/\mathrm{d}g^N < 0$, $B_2/\mathrm{d}g^N < 0$, and $\omega_2^1 < 0$. As we can see from (186a), investment dynamics are the result of two opposite forces. If the initial investment flow is positive, it must turn to be negative at time $\tilde{t}$ along the trajectory:

$$
\tilde{t} = \frac{1}{\mu_1 - \mu_2}\ln\left[-\frac{\mu_2 B_2/\mathrm{d}g^N}{\mu_1 B_1/\mathrm{d}g^N}\right], \tag{187}
$$

where the term in square brackets is less than one under the condition that the initial investment flow is positive (see (182), otherwise the investment dynamics are monotonic.

The current account dynamics over period 1 is described by the following equation:

$$
ca(t) = \left[\tilde{p}e^{-\mu_2(\mathcal{T}-t)}\left(1-e^{-\mu_1(\mathcal{T}-t)}\right) + \mu_1 \frac{N_1}{\mu_1 - r^*}\frac{B_1}{\mathrm{d}g^N}e^{\mu_1 t}\right]\mathrm{d}g^N < 0. \tag{188}
$$

We turn now to the analysis of transitional dynamics over the stable period 2. By making use of standard methods, the adjustments of the stock of physical capital, the real exchange rate and the stock of international assets are driven by the following equations:

$$
\dot{K}(t) = \mu_1 \frac{B_1'}{\mathrm{d}g^N}\mathrm{d}g^N e^{\mu_1 t} > 0, \tag{189a}
$$

$$
\dot{p}(t) = \mu_1 \omega_2^1 \frac{B_1'}{\mathrm{d}g^N}\mathrm{d}g^N e^{\mu_1 t} < 0, \tag{189b}
$$

$$
\dot{n}(t) = \mu_1 \frac{N_1}{\mu_1 - r^\star}\frac{B_1'}{\mathrm{d}g^N}\mathrm{d}g^N e^{\mu_1 t} < 0. \tag{189c}
$$

Evaluate (189c) at time t^+, and calculate $\mathrm{d}ca(\mathcal{T}) = ca(\mathcal{T}^+) - ca(\mathcal{T}^-)$, we can see that the current account is continuous in the neighborhood of time $\mathcal{T}$. Thus we have $ca(\mathcal{T}^-) = ca(\mathcal{T}^+)$. Performing the same procedure of investment, we obtain:

$$\frac{\mathrm{d}I(\mathcal{T})}{\mathrm{d}g^N} = -\mu_2 \frac{B_2}{\mathrm{d}g^N} e^{\mu_2 \mathcal{T}} = 1. \tag{190}$$

When the policy is removed at time $\mathcal{T}$, i. e. government spending falls by an amount equals to $\mathrm{d}g^N(\mathcal{T}) \equiv g_2^N - g_1^N \equiv -\mathrm{d}g^N$, investment must rise to guarantee that the market-clearing condition holds at time $\mathcal{T}$.

case $k^T > k^N$

Life after an unanticipated permanent fiscal expansion, an unexpected transitory rise in government spending on the non traded good leaves unaffected the real exchange rate both in the short-run and in the long-run. To evaluate the investment dynamics, we differentiate the solution for $K(t)$ given by (100a) with respect to time, evaluate the resulting expression at time $t = 0$, and then differentiate with respect to g^N, keeping in mind that $B_2/\mathrm{d}g^N = 0$ if $k^T > k^N$:

$$\begin{aligned}
\left.\frac{\mathrm{d}I(0)}{\mathrm{d}g^N}\right|_{temp} &= \mu_1 \frac{B_1}{\mathrm{d}g^N} = -\mu_1 \frac{1}{\mu_1}\left[(1-\alpha_c)\left(1 - e^{-r^\star \mathcal{T}}\right) + e^{-r^\star \mathcal{T}}\right], \\
&= \left.\frac{\mathrm{d}I(0)}{\mathrm{d}g^N}\right|_{perm}\left(1 - e^{-r^\star \mathcal{T}}\right) - e^{-r^\star \mathcal{T}} < 0.
\end{aligned} \tag{191}$$

Applying standard methods, the initial response of the current account following an unanticipated temporary fiscal expansion on the non traded good is given by:

$$\begin{aligned}
\left.\frac{\mathrm{d}ca(0)}{\mathrm{d}g^N}\right|_{temp} &= r^\star \left\{ -\left.\frac{\mathrm{d}\tilde{n}_1}{\mathrm{d}g^N}\right|_{temp} - \frac{N_1}{\mu_1 - r^\star}\frac{B_1}{\mathrm{d}g^N} \right\} + \mu_1 \frac{N_1}{\mu_1 - r^\star}\frac{B_1}{\mathrm{d}g^N}, \\
&= \tilde{p}(1-\alpha_c)\left(1 - e^{-r^\star \mathcal{T}}\right) > 0,
\end{aligned} \tag{192}$$

where $\mu_1 \frac{N_1}{\mu_1 - r^\star}\frac{B_1}{\mathrm{d}g^N} = \tilde{p}\left[(1-\alpha_c)\left(1 - e^{-r^\star \mathcal{T}}\right) + e^{-r^\star \mathcal{T}}\right]$ and we simplified the term in square braces as follows:

$$\begin{aligned}
& -\left.\frac{\mathrm{d}\tilde{n}_1}{\mathrm{d}g^N}\right|_{temp} - \frac{N_1}{\mu_1 - r^\star}\frac{B_1}{\mathrm{d}g^N} \\
&= -\left\{\left[\left(v_{\bar{\lambda}} - \frac{N_1}{\mu_1 - r^\star}K_{\bar{\lambda}}\right)\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} + \left(v_{g^N} - \frac{N_1}{\mu_1 - r^\star}K_{g^N}\right)\right]\right\}, \\
&= \frac{\lambda_{g^N}}{\lambda_n} e^{-r^\star \mathcal{T}} = -\frac{\tilde{p}}{r^\star} e^{-r^\star \mathcal{T}} < 0.
\end{aligned} \tag{193}$$

We investigate the dynamics of the stocks of physical capital and traded bonds by taking the time derivative of formal solutions prevailing over period 1:

$$\begin{aligned} I(t) &= \dot{K}(t) = \mu_1 \frac{B_1}{\mathrm{d}g^N} \mathrm{d}g^N e^{\mu_1 t}, \\ &= -\mu_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) \mathrm{d}g^N e^{\mu_1 t} - e^{-r^\star \mathcal{T}} \mathrm{d}g^N e^{\mu_1 t} < 0, \end{aligned} \tag{194}$$

and

$$\begin{aligned} ca(t) &= -r^\star \left[\left(v\left(\bar{\lambda}, g_1^N\right) - v\left(\lambda_0, g_0^N\right)\right) + \frac{N_1}{\mu_1 - r^\star} \frac{B_1}{\mathrm{d}g^N} \right] \mathrm{d}g^N e^{r^\star t} + \mu_1 \frac{N_1}{\mu_1 - r^\star} \frac{B_1}{\mathrm{d}g^N} \mathrm{d}g^N e^{\mu_1 t}, \\ &= \tilde{p} \left[(1 - \alpha_c) \left(1 - e^{-r^\star \mathcal{T}}\right) e^{\mu_1 t} - e^{-r^\star \mathcal{T}} \left(e^{r^\star t} - e^{\mu_1 t}\right) \right] \mathrm{d}g^N \gtrless 0. \end{aligned} \tag{195}$$

There exists a time $t = \acute{t}$ such that the current account changes of sign:

$$\acute{t} = -\frac{1}{\mu_2} \ln \left[\frac{e^{-r^\star \mathcal{T}}}{(1 - \alpha_c)\left(1 - e^{-r^\star \mathcal{T}}\right) + e^{-r^\star \mathcal{T}}} \right], \tag{196}$$

where the term in square brackets is positive and lower than one. Over period 1, the current account improves first while the negative investment flow more than outweighs the *smoothing* effect. At time $\acute{t}$, these two effects cancel each other and after this date, the current account deteriorates as the smoothing behavior predominates, such that $ca\left(\mathcal{T}^-\right) < 0$. To see it more formally, we evaluate (195) at time $\mathcal{T}^-$:

$$ca\left(T^-\right) = \tilde{p} e^{\mu_1 \mathcal{T}} \left[\left(1 - e^{-\mu_1 \mathcal{T}}\right) - \alpha_c \left(1 - e^{-r^\star \mathcal{T}}\right) \right] \mathrm{d}g^N < 0. \tag{197}$$

At time $\mathcal{T}^-$, the investment flow is also negative:

$$I\left(\mathcal{T}^-\right) = -e^{-\mu_2 \mathcal{T}} \left[1 - (1 - \alpha_c) \left(1 - e^{r^\star \mathcal{T}}\right) \right] < 0. \tag{198}$$

We have now to compare the slope of the trajectory after a transitory fiscal expansion over period $0 \leq t < \acute{t}$ in the (K, n)-space with the slope of the trajectory after a permanent fiscal expansion:

$$\begin{aligned} \frac{\mathrm{d}n(t)}{\mathrm{d}K(t)} &= \frac{-\tilde{p} e^{-r^\star (\mathcal{T} - t)} + \mu_1 \frac{N_1}{\mu_1 - r^\star} \frac{B_1}{\mathrm{d}g^N} e^{\mu_1 t}}{\mu_1 \frac{B_1}{\mathrm{d}g^N} e^{\mu_1 t}}, \\ &= -\frac{\tilde{p} \left\{ \left[(1 - \alpha_c) \left(1 - e^{-r^\star \mathcal{T}}\right) + e^{-r^\star \mathcal{T}} \right] e^{\mu_1 t} - e^{-r^\star (\mathcal{T} - t)} \right\}}{\left[(1 - \alpha_c) \left(1 - e^{-r^\star \mathcal{T}}\right) + e^{-r^\star \mathcal{T}} \right] e^{\mu_1 t}}, \end{aligned} \tag{199}$$

where we have substituted the expression of the constant $B_1/\mathrm{d}g^N$. Over period $0 \leq \acute{t} < t$, the numerator is positive and the denominator is negative. Thus the slope of the trajectory is negative in the (K, n)-space. Comparing the terms in numerator and in denominator of (199), it is straightforward to show that the slope in absolute terms is lower than $\tilde{p}$. Therefore, the slope is negative and lower (in absolute terms) than the slope of the trajectory after a permanent fiscal expansion (equal to $-\tilde{p}$).

We turn now to the investigation of transitional dynamics of key macroeconomic variables over the stable period, say period 2. By adopting the standard procedure, we get:

$$\begin{aligned} I(t) &= \dot{K}(t) = \mu_1 \frac{B_1'}{\mathrm{d}g^N} \mathrm{d}g^N e^{\mu_1 t} > 0 && \text{(200a)} \\ ca(t) &= \dot{n}(t) = \mu_1 \frac{N_1}{\mu_1 - r^\star} \frac{B_1'}{\mathrm{d}g^N} \mathrm{d}g^N e^{\mu_1 t} < 0. && \text{(200b)} \end{aligned}$$

Since the period 2 is a stable period, the dynamics are monotonic. If we can determine the sign of (200) at time $t = \mathcal{T}^+$, we are able to evaluate the transitional dynamics over the entire period:

$$\begin{aligned} I\left(\mathcal{T}^+\right) &= -\left[(1-\alpha_c)\left(e^{\mu_1 \mathcal{T}} - e^{-\mu_2 \mathcal{T}}\right) - \left(1 - e^{-\mu_2 \mathcal{T}}\right)\right] \mathrm{d}g^N > 0, && \text{(201a)} \\ ca\left(\mathcal{T}^+\right) &= \tilde{p}\left[(1-\alpha_c)\left(e^{\mu_1 \mathcal{T}} - e^{-\mu_2 \mathcal{T}}\right) - \left(1 - e^{-\mu_2 \mathcal{T}}\right)\right] \mathrm{d}g^N < 0. && \text{(201b)} \end{aligned}$$

From (197) and (201b), we deduce that the current account is continuous in the neighborhood of $\mathcal{T}$, such that $ca\left(\mathcal{T}^-\right) = ca\left(\mathcal{T}^+\right) < 0$. At the opposite, from (198) and (201a), we see that investment is not continuous in the neighborhood of T since at this date, it must clear the non tradable market. To see it formally, we write the non tradable clearing market condition at time $\mathcal{T}^-$ and at time $\mathcal{T}^+$:

$$\begin{aligned} I\left(\mathcal{T}^-\right) &= Y^N\left[K\left(\mathcal{T}^-\right), e\left(\mathcal{T}^-\right)\right] - p_c'\left[e\left(\mathcal{T}^-\right)\right] c\left[\lambda\left(\mathcal{T}^-\right), e\left(\mathcal{T}^-\right)\right] - g_1^N < 0, && \text{(202a)} \\ I\left(\mathcal{T}^+\right) &= Y^N\left[K\left(\mathcal{T}^+\right), e\left(\mathcal{T}^+\right)\right] - p_c'\left[e\left(\mathcal{T}^+\right)\right] c\left[\lambda\left(\mathcal{T}^+\right), e\left(\mathcal{T}^+\right)\right] - g_2^N > 0, && \text{(202b)} \end{aligned}$$

where $g_2^N = g_0^N$. Goods market equilibrium is subject to two discrete perturbations: one at time $t = 0$ when the government raises the public spending, the other at time $t = \mathcal{T}$ when the policy is permanently removed. Since capital is a predetermined variable, it cannot jump at time $t = 0$ neither at time $t = \mathcal{T}$. In addition, the marginal utility of wealth jumps at time $t = 0$ and remains constant from thereon. So we get $\bar{\lambda} = \lambda\left(\mathcal{T}^-\right) = \lambda\left(\mathcal{T}^+\right)$. Finally, when the tradable good sector is relatively more capital intensive, a rise in government spending leaves unaffected the real exchange rate both in the sort-run and in the long-run, such that $\tilde{p} = e\left(\mathcal{T}^-\right) = e\left(\mathcal{T}^+\right)$. With output constrained at time $\mathcal{T}$ by the capital stock and by the real exchange rate, it therefore follows from (202) that for the market-clearing condition to hold, we must have

$$\mathrm{d}I\left(\mathcal{T}\right) = \mathrm{d}\dot{K}\left(\mathcal{T}\right) = -\mathrm{d}g^N\left(\mathcal{T}\right) = \mathrm{d}g^N > 0, \tag{203}$$

where $\mathrm{d}g^N(\mathcal{T}) \equiv g_2^N - g_1^N \equiv g_0^N - g_1^N \equiv -\mathrm{d}g^N$. Thus, the non traded goods market equilibrium is maintained though the investment in physical capital, government expenditure is reduced to its original level, the investment flow changes of sign and turns to be positive as a greater part of the non tradable production (Y^N) may be allocated to investment (I) since the global consumption ($c^N + g^N$) falls.

F Savings

Formal Solution for Financial Wealth

The law of motion for financial wealth ($S(t) = \dot{a}(t)$) is given by:

$$\dot{a}(t) = r^\star a(t) + w\left(p(t)\right) - p_c\left(p(t)\right) c\left(\bar{\lambda}, p(t)\right) - Z, \tag{204}$$

with $Z = g^T + pg^N$.

The linearized version of (204) writes as follows:

$$\begin{aligned}\dot{a}(t) &= r^\star\left(a(t) - \tilde{a}\right) + \left[w_p - \left(p_c'\tilde{c} + p_c c_p\right) - g^N\right]\left(p(t) - \tilde{p}\right), \\ &= r^\star\left(a(t) - \tilde{a}\right) + M_1\left(p(t) - \tilde{p}\right),\end{aligned} \tag{205}$$

with M_1 given by

$$\begin{aligned}M_1 &= -\left[\tilde{k}^T\mu_2 + \tilde{c}^N\left(1 - \sigma_c\right) - g^N\right] < 0, \\ &= -\left(\tilde{K}\mu_2 - \sigma_c\tilde{c}^N\right) < 0 \quad \text{if} \quad k^N > k^T, \qquad (206\text{a}) \\ &= 0 \quad \text{if} \quad k^T > k^N, \qquad (206\text{b})\end{aligned}$$

where we used the fact that $\tilde{k}^T\mu_2 + \tilde{Y}^N + g^N = \tilde{K}\mu_2$. Since $\sigma_c < 1$ as empirical evidence suggest, the sign of M_1 given by (206a) is negative.

The general solution for the stock of financial wealth writes as follows:

$$a(t) = \tilde{a} + \left[\left(a_0 - \tilde{a}\right) - \frac{M_1\omega_2^1}{\mu_1 - r^\star}B_1\right]e^{r^\star t} + \frac{M_1\omega_2^1}{\mu_1 - r^\star}B_1 e^{\mu_1 t}. \tag{207}$$

where we used the fact that $\omega_2^2 = 0$. Invoking the transversality condition (8), we obtain the stable solution for financial wealth:

$$a(t) = \tilde{a} + \frac{M_1\omega_2^1}{\mu_1 - r^\star}B_1 e^{\mu_1 t}, \tag{208}$$

and the intertemporal solvency condition

$$\tilde{a} - a_0 = \frac{M_1\omega_2^1}{\mu_1 - r^\star}\left(\tilde{K} - K_0\right). \tag{209}$$

Differentiating (209) w.r.t g^i ($i = T, N$), long-term changes of financial wealth when $k^N > k^T$ are given by:

$$\frac{\mathrm{d}\tilde{a}}{\mathrm{d}g^T} = \frac{\omega_2^1}{\mu_2}\left(\tilde{K}\mu_2 - \sigma_c\tilde{c}^N\right)\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T} > 0, \quad (210a)$$

$$\frac{\mathrm{d}\tilde{a}}{\mathrm{d}g^N} = \frac{\omega_2^1}{\mu_2}\left(\tilde{K}\mu_2 - \sigma_c\tilde{c}^N\right)\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N} < 0. \quad (210b)$$

When $k^T > k^N$, long-term changes of financial wealth are given by:

$$\frac{\mathrm{d}\tilde{a}}{\mathrm{d}g^T} = \frac{\mathrm{d}\tilde{a}}{\mathrm{d}g^N} = 0. \quad (211)$$

Another way to compute long-term changes of financial wealth is to first linearize its expression defined by the sum of internationally traded bonds and domestic shares : $a(t) = n(t) + p(t)K(t)$. The linearization in the neighborhood of the steady-state yields:

$$a(t) - \tilde{a} = (n(t) - \tilde{n}) + \tilde{p}\left(K(t) - \tilde{K}\right) + \tilde{K}\left(p(t) - \tilde{p}\right)$$

Evaluate at time $t = 0$:

$$a(0) - \tilde{a} = (n(0) - \tilde{n}) + \tilde{p}\left(K(0) - \tilde{K}\right) + \tilde{K}\left(p(0) - \tilde{p}\right)$$

Using the fact that $a(0) = a_0$, $K(0) = K_0$ and $p(0) - \tilde{p} = \omega_2^1 B_1$, we get

$$a_0 - \tilde{a} = \frac{N_1}{\mu_1 - r^\star}\left(K_0 - \tilde{K}\right) + \tilde{p}\left(K_0 - \tilde{K}\right) + \tilde{K}\omega_2^1\left(K_0 - \tilde{K}\right).$$

Assuming that the small open economy starts from an initial steady-state, such that $a_0 = \tilde{a}_0$ and $K_0 = \tilde{K}_0$

$$\tilde{a}_0 - \tilde{a} = \frac{N_1}{\mu_1 - r^\star}\left(\tilde{K}_0 - \tilde{K}\right) + \tilde{p}_0\left(\tilde{K}_0 - \tilde{K}\right) + \tilde{K}_0\omega_2^1\left(\tilde{K}_0 - \tilde{K}\right)B_1.$$

Keeping in mind that $\tilde{x}_0 - \tilde{x} = -\mathrm{d}\tilde{x}$, we obtain the long term change in the financial wealth:

$$\mathrm{d}\tilde{a} = \left[\frac{N_1}{\mu_1 - r^\star} + \tilde{p}_0 + \omega_2^1\tilde{K}_0\right]\mathrm{d}\tilde{K},$$

with

$$\left[\frac{N_1}{\mu_1 - r^\star} + \tilde{p} + \omega_2^1\tilde{K}\right] = \frac{\omega_2^1}{\mu_2}\left(\tilde{K}\mu_2 - \sigma_c\tilde{c}^N\right) < 0.$$

Differentiating (208) w.r.t, we get the dynamics of savings:

$$S(t) = \dot{a}(t) = \mu_1\frac{M_1\omega_2^1}{\mu_1 - r^\star}\frac{B_1}{\mathrm{d}g^T}\mathrm{d}g^T e^{\mu_1 t} > 0, \quad (212a)$$

$$S(t) = \dot{a}(t) = \mu_1\frac{M_1\omega_2^1}{\mu_1 - r^\star}\frac{B_1}{\mathrm{d}g^N}\mathrm{d}g^N e^{\mu_1 t} < 0, \quad (212b)$$

where $\frac{B_1}{\mathrm{d}g^T} = -\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^T} > 0$ and $\frac{B_1}{\mathrm{d}g^N} = -\frac{\mathrm{d}\tilde{K}}{\mathrm{d}g^N} < 0$.

An Unanticipated Temporary Rise in Government Spending

We now evaluate the transitional dynamics of saving after an unanticipated temporary shock, $\mathrm{d}g_i$ $(i = T, N)$.

$k^N > k^T$

Over the unstable period 1, saving evolves as follows:

$$S(t) = \dot{a}(t) = r^\star \left[(a_0 - \tilde{a}_1) - \frac{M_1 \omega_2^1}{\mu_1 - r^\star} B_1 \right] e^{r^\star t} + \mu_1 \frac{M_1 \omega_2^1}{\mu_1 - r^\star} B_1 e^{\mu_1 t}, \tag{213}$$

with

$$(a_0 - \tilde{a}_1) = (n_0 - \tilde{n}_1) + \tilde{p}_0 \left(K_0 - \tilde{K}_1 \right) + K_0 \left(p_0 - \tilde{p}_1 \right). \tag{214}$$

Over the stable period 2, saving evolves as follows:

$$S(t) = \dot{a}(t) = \mu_1 \frac{M_1 \omega_2^1}{\mu_1 - r^\star} B_1' e^{\mu_1 t}, \tag{215}$$

To compute the long-term changes of financial wealth, we linearize its expression, i. e. $a(t) = n(t) + p(t)K(t)$ in the neighborhood of the final steady-state:

$$a(t) - \tilde{a}_2 = (n(t) - \tilde{n}_2) + \tilde{p} \left(K(t) - \tilde{K}_2 \right) + \tilde{K} \left(p(t) - \tilde{p}_2 \right)$$

Then we evaluate at time $t = 0$:

$$a_0 - \tilde{a}_2 = (n_0 - \tilde{n}_2) + \tilde{p}_0 \left(K_0 - \tilde{K}_2 \right) + \tilde{K}_0 \left(p(0) - \tilde{p}_2 \right),$$

where we used the fact that $a(0) = a_0$, $n(0) = n_0$, $K(0) = K_0$ and assumed that the small open economy starts initially from the steady-state, i. e. $a_0 = \tilde{a}_0 = \tilde{a}$, $n_0 = \tilde{n}_0 = \tilde{n}$, $K_0 = \tilde{K}_0 = \tilde{K}$. Substituting $p(0) - \tilde{p}_2 = \omega_2^1 B_1$ into the expression above and differentiating w.r.t g^i $(i = T, N)$, long-term changes of financial wealth are given by:

$$\left.\frac{\mathrm{d}\tilde{a}}{\mathrm{d}g^T}\right|_{temp} = \left(v_{\bar{\lambda}} + \tilde{p} K_{\bar{\lambda}} \right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp} < 0, \tag{216a}$$

$$\left.\frac{\mathrm{d}\tilde{a}}{\mathrm{d}g^N}\right|_{temp} = \left(v_{\bar{\lambda}} + \tilde{p} K_{\bar{\lambda}} \right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} < 0, \tag{216b}$$

with

$$\left(v_{\bar{\lambda}} + \tilde{p} K_{\bar{\lambda}} \right) = -\frac{\sigma_c p_c \tilde{c}}{\bar{\lambda} r^\star} < 0. \tag{217}$$

$k^T > k^N$

Since $\omega_2^1 = 0$ whenever the traded good sector is relatively more capital intensive, and because $B_2/\mathrm{d}g^i = 0$, the transitional dynamics for saving degenerate and the financial wealth jumps immediately to its new steady-state level.

Adopting a similar procedure than previously (i. e. in the case $k^N > k^T$), we can calculate the long-term changes of financial wealth as follows:

$$\left.\frac{\mathrm{d}\tilde{a}}{\mathrm{d}g^T}\right|_{temp} = \left(v_{\bar{\lambda}} + \tilde{p}K_{\bar{\lambda}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^T}\right|_{temp} < 0, \tag{218a}$$

$$\left.\frac{\mathrm{d}\tilde{a}}{\mathrm{d}g^N}\right|_{temp} = \left(v_{\bar{\lambda}} + \tilde{p}K_{\bar{\lambda}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^N}\right|_{temp} < 0, \tag{218b}$$

with

$$v_{\bar{\lambda}} + \tilde{p}K_{\bar{\lambda}} = -\frac{\sigma_c p_c \tilde{c}}{\bar{\lambda} r^{\star}} < 0. \tag{219}$$